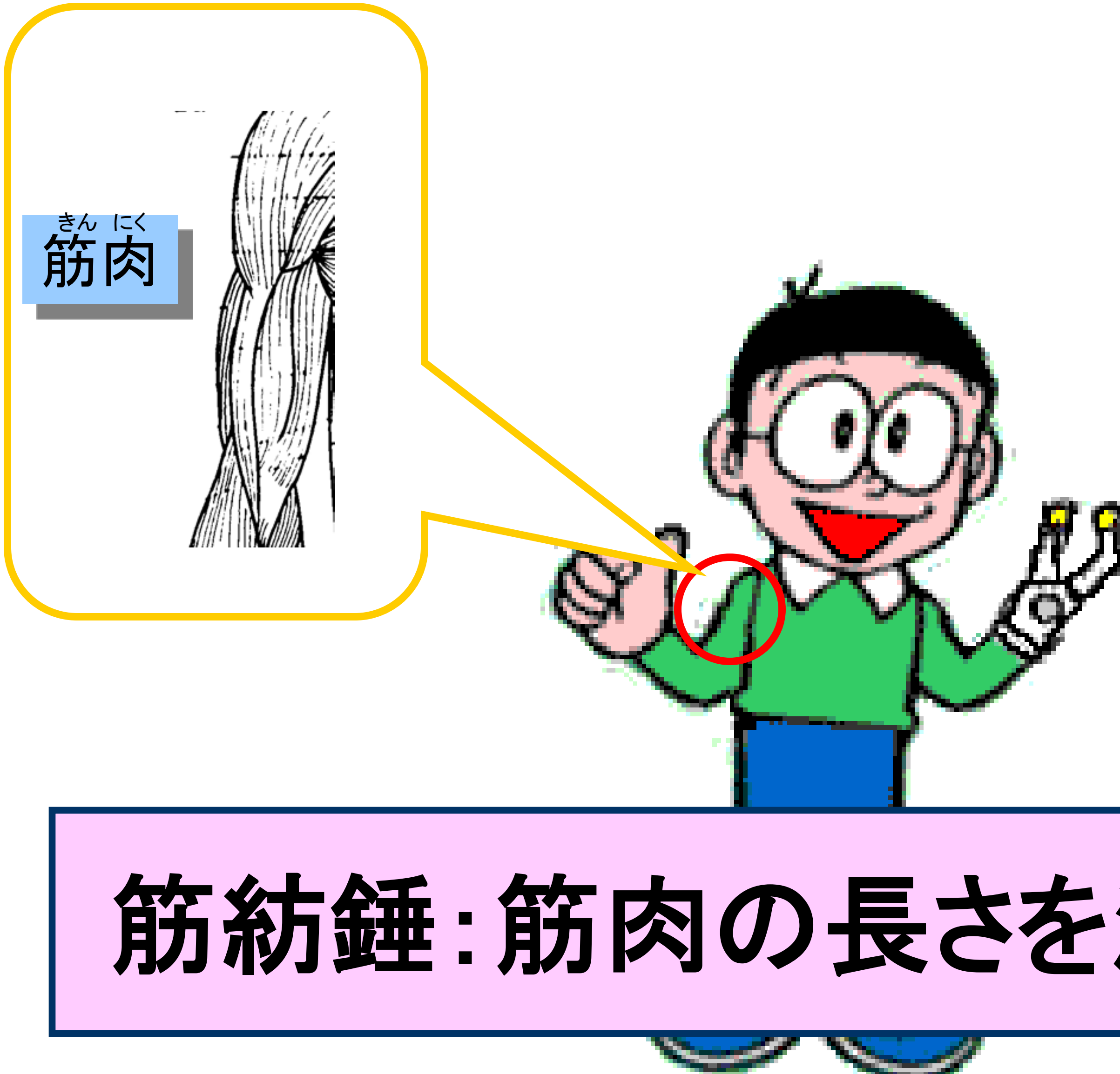
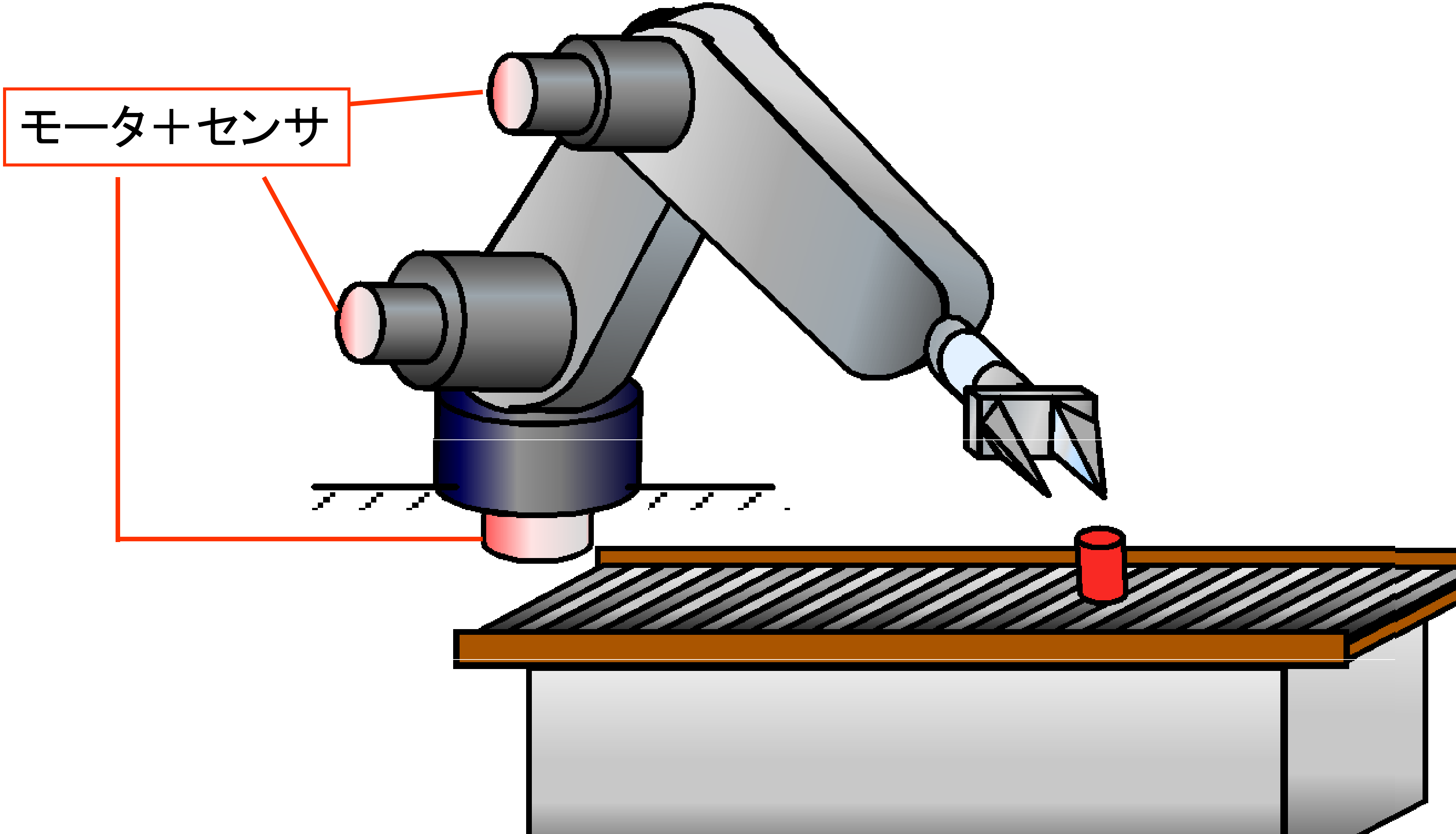
機構学
Part6：ロボットの運動学

金子真

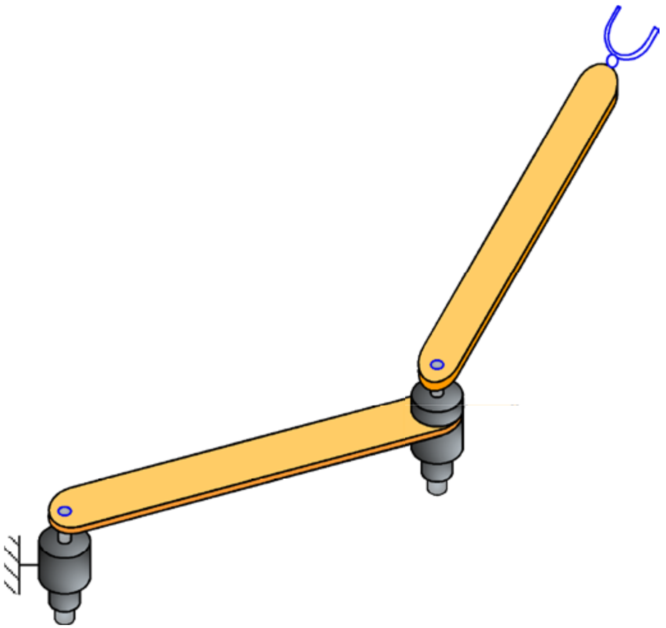


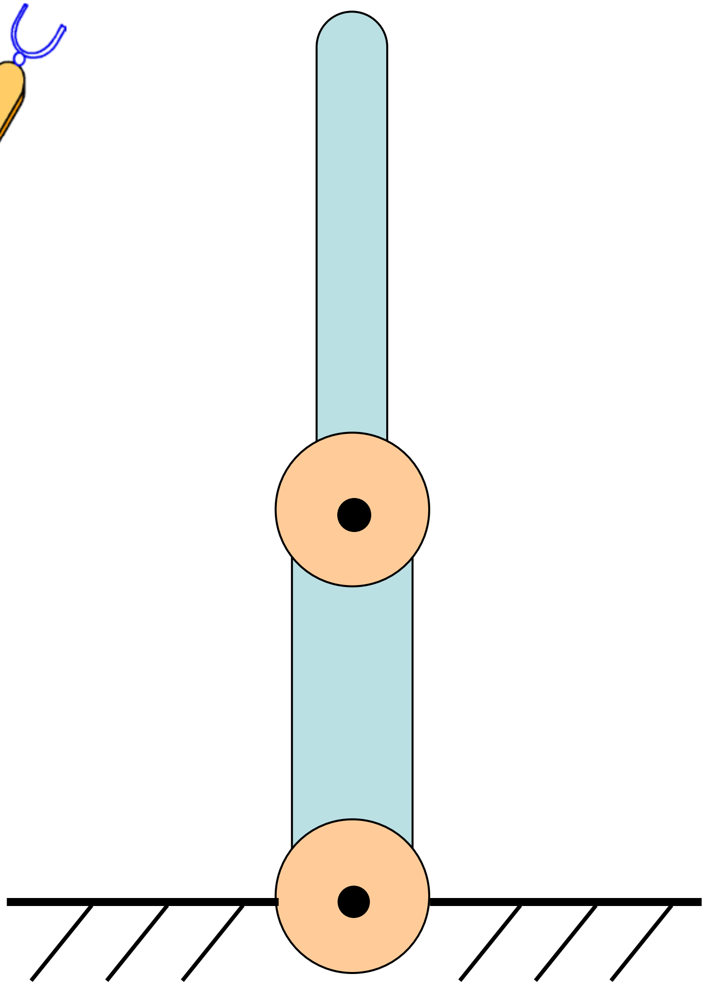
筋紡錘：筋肉の長さを測るセンサ

ロボットの運動学



関節にモータがついている場合の角度の取り方





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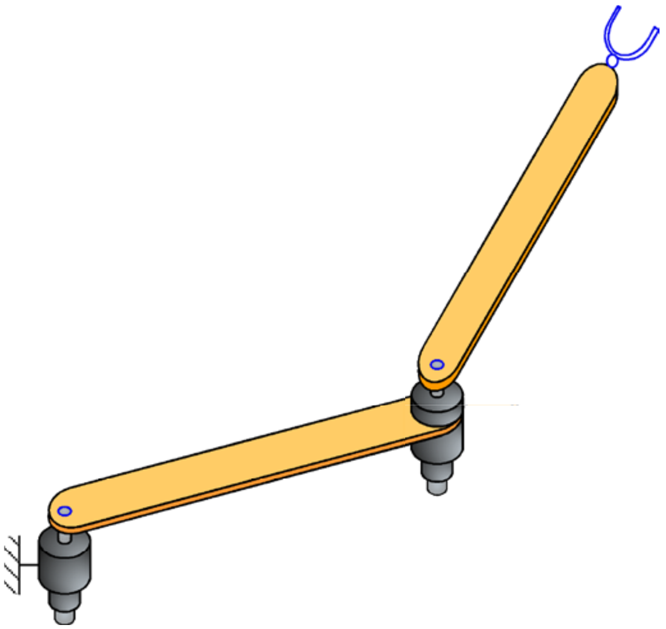

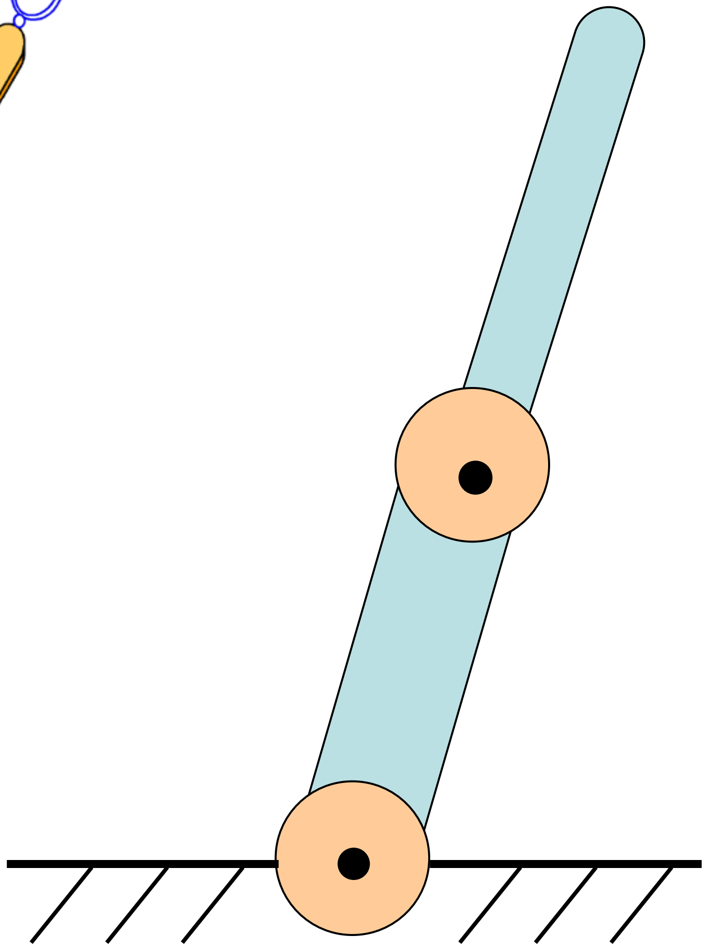

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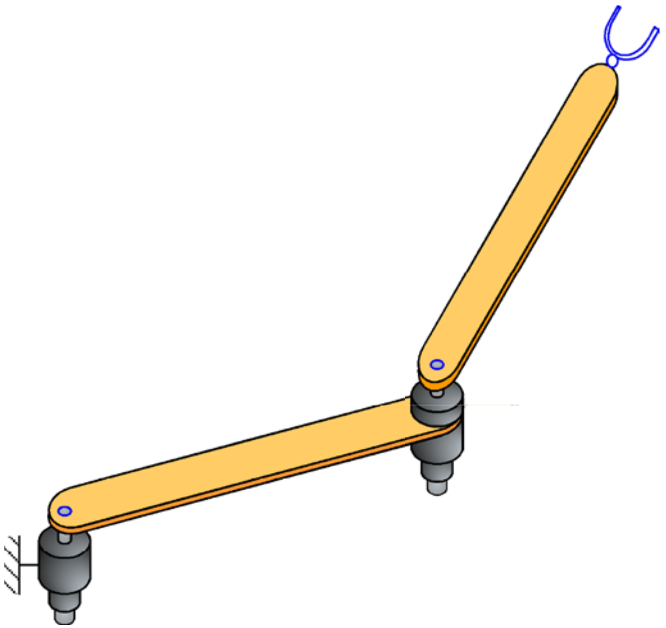
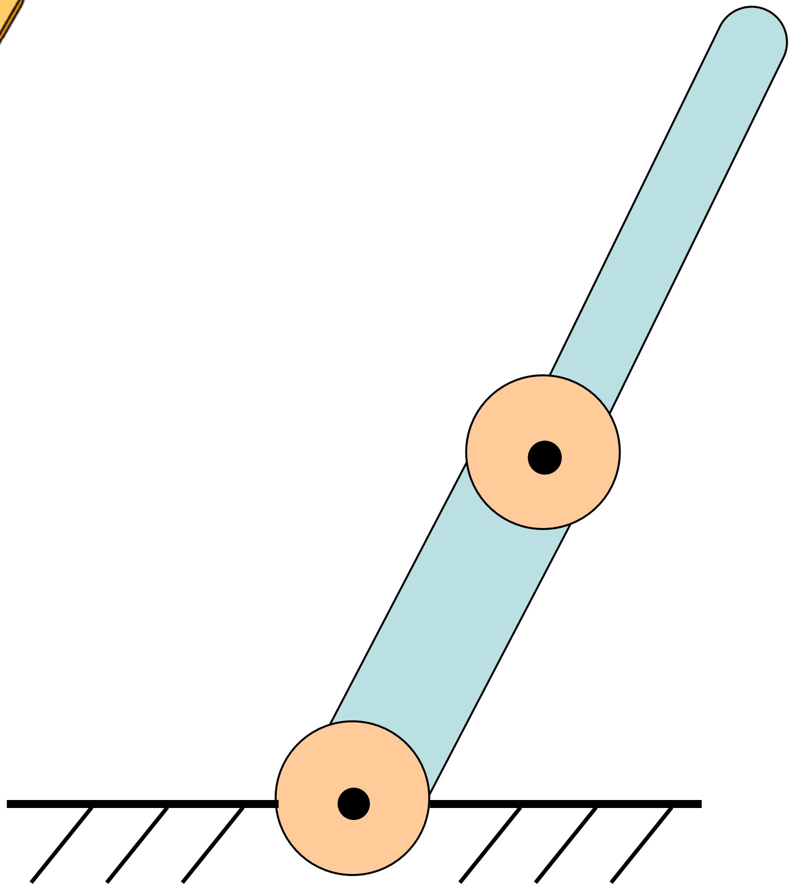
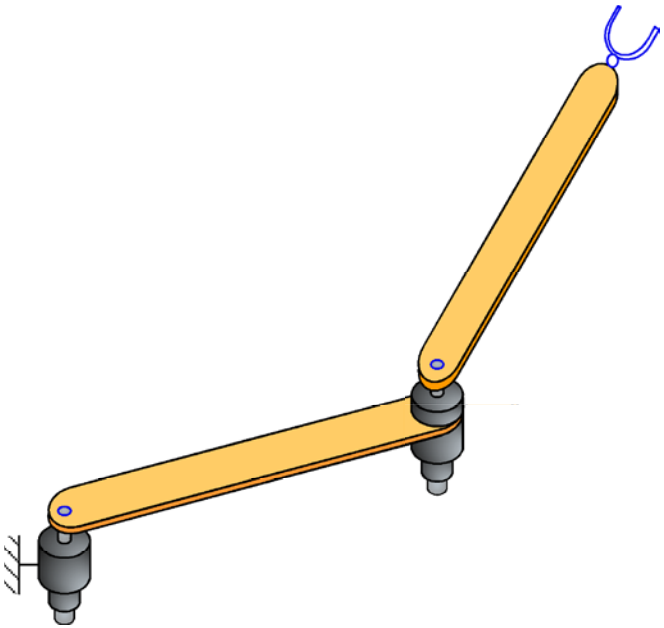
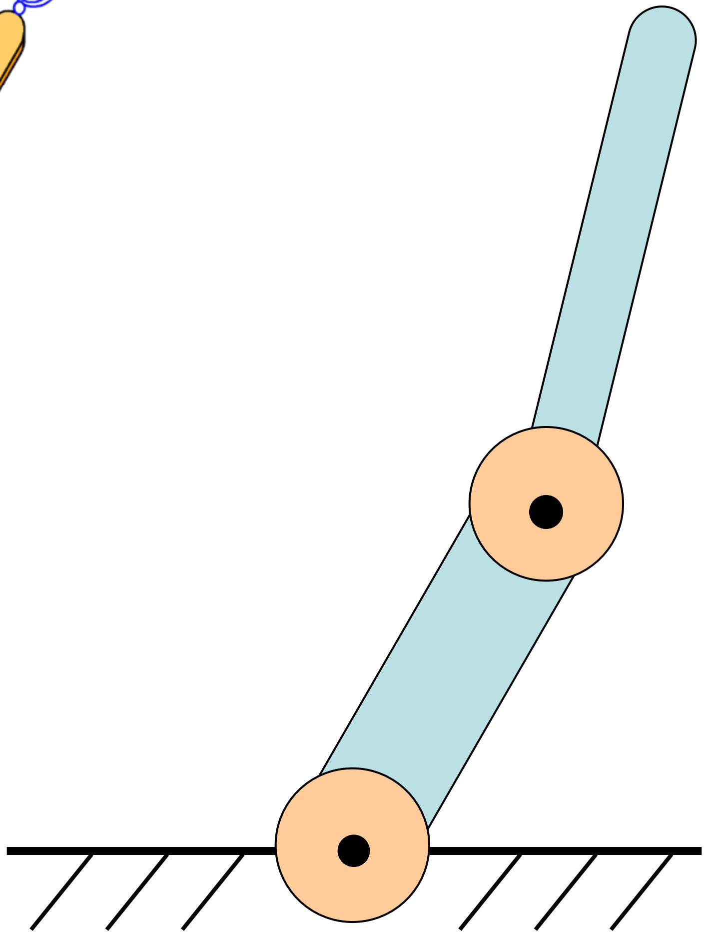


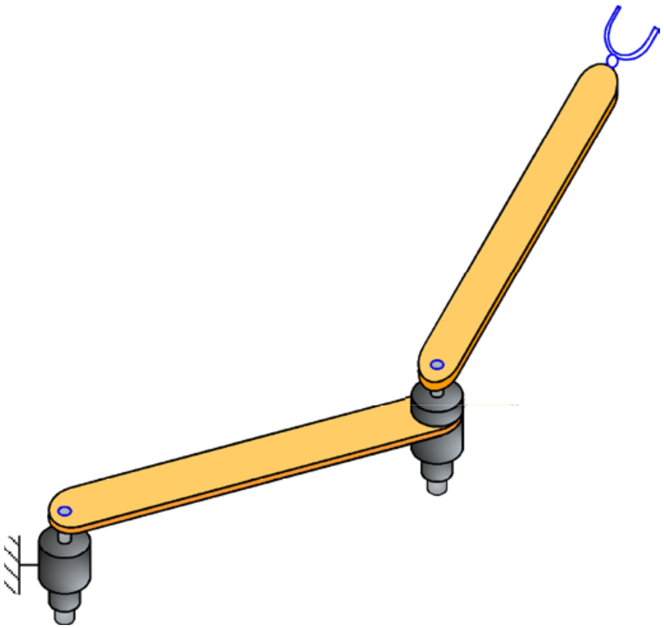


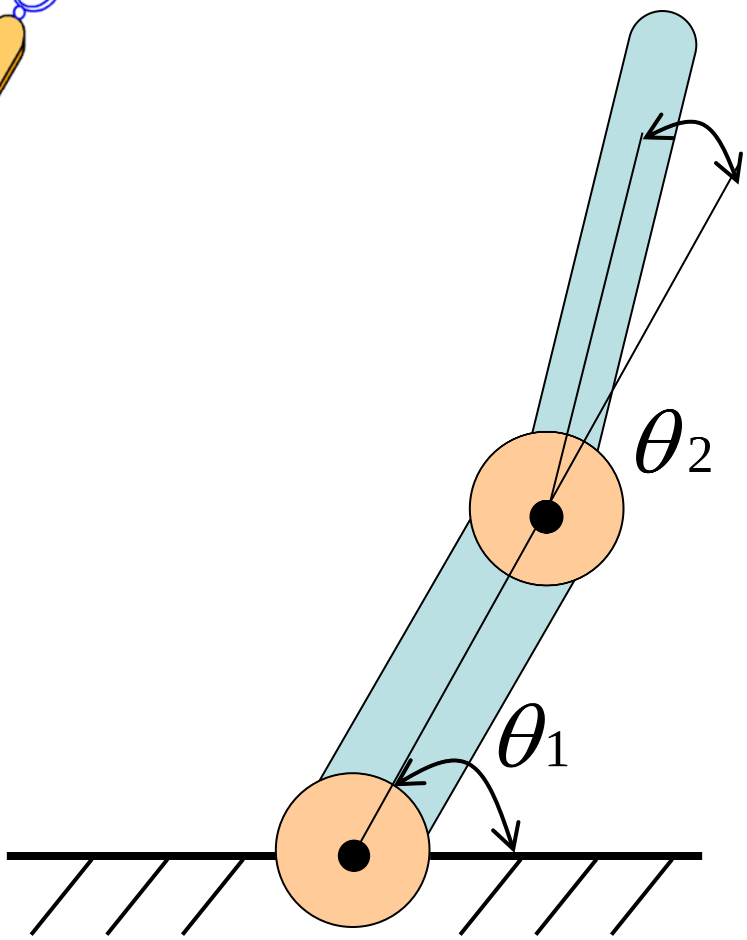
関節にモータがついている場合の角度の取り方



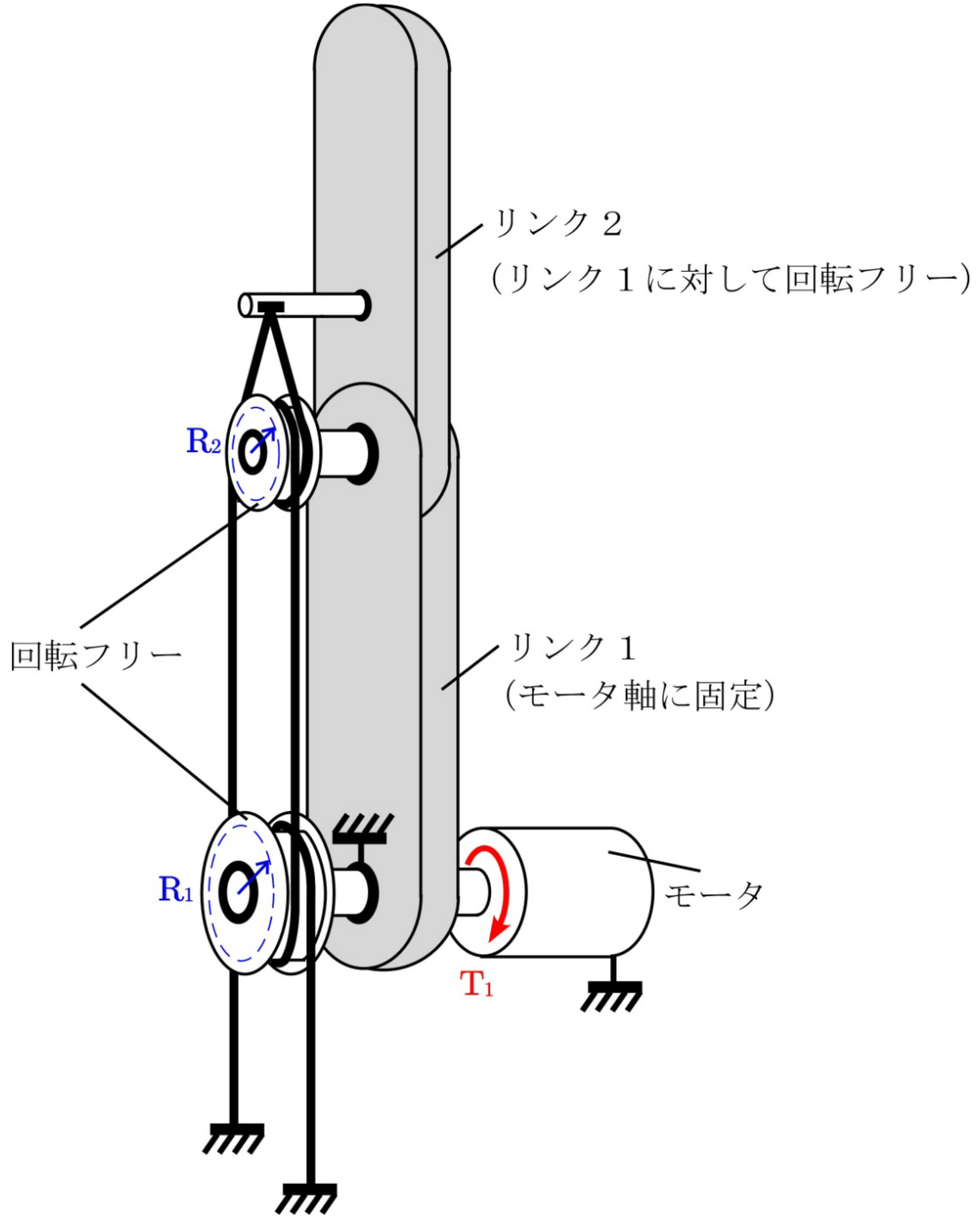


関節にモータがついている場合の角度の取り方

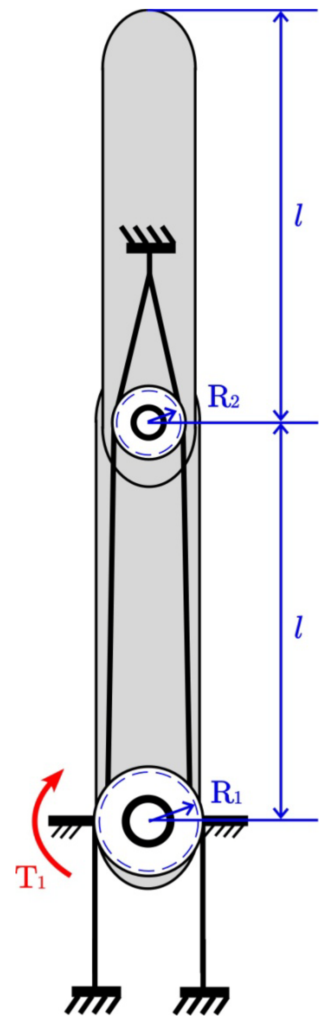




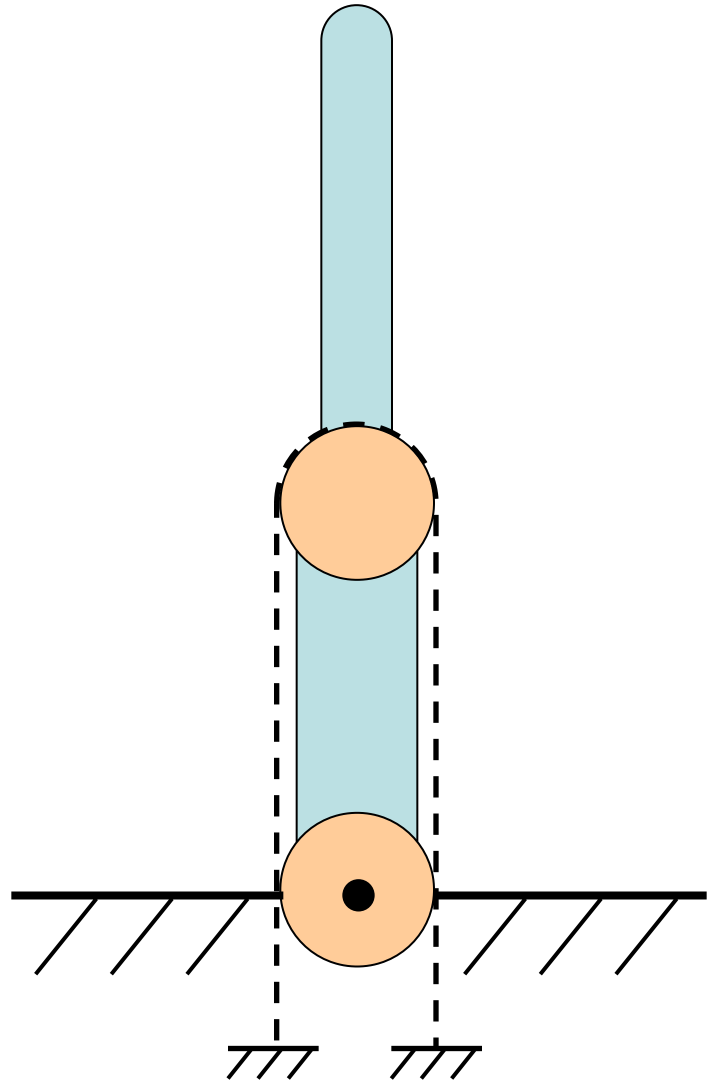
ワイヤ駆動式ロボット



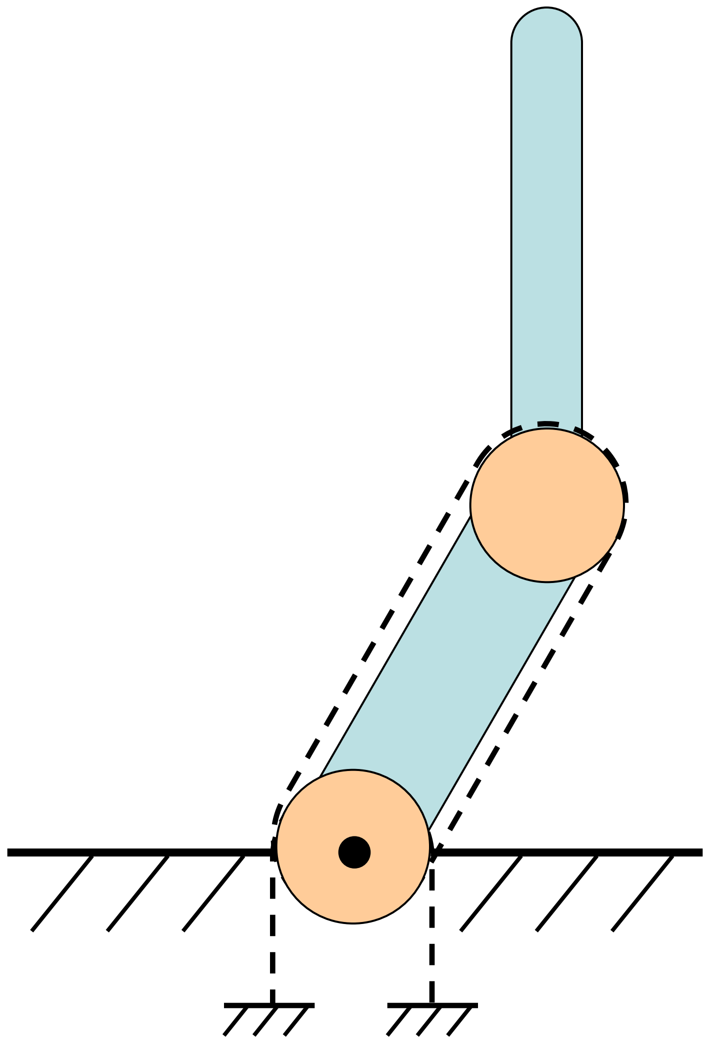
ワイヤ駆動式ロボット



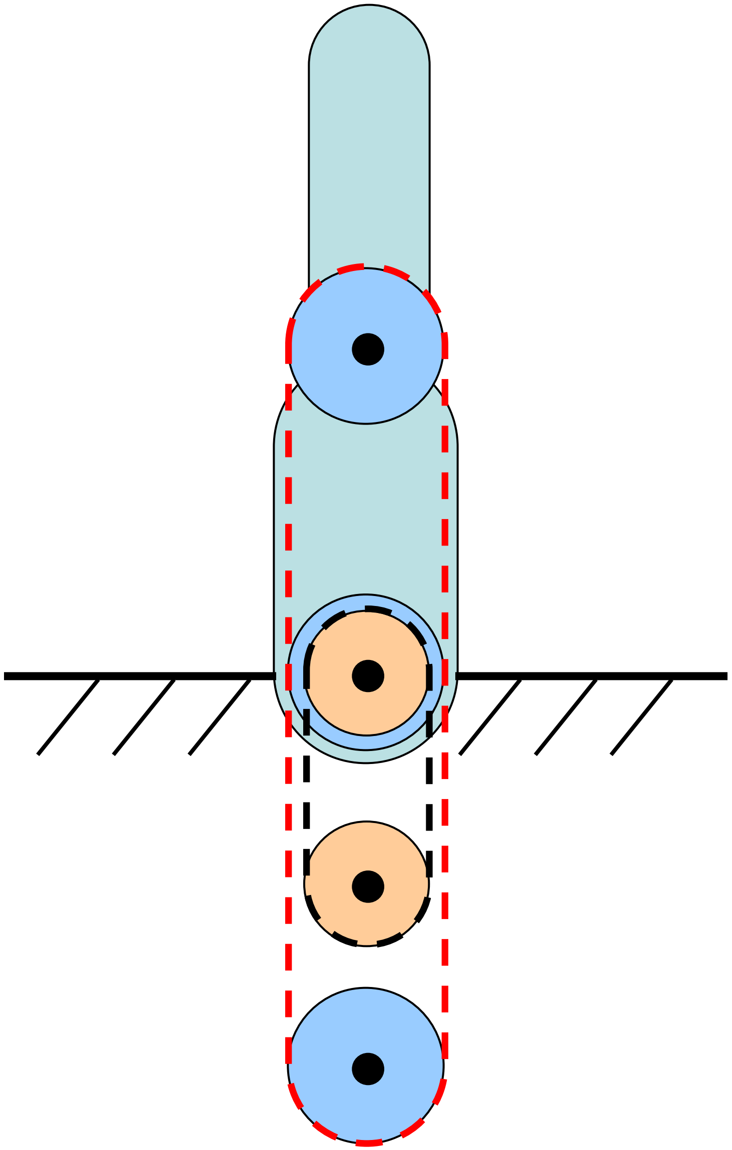
ワイヤプーリ機構の場合



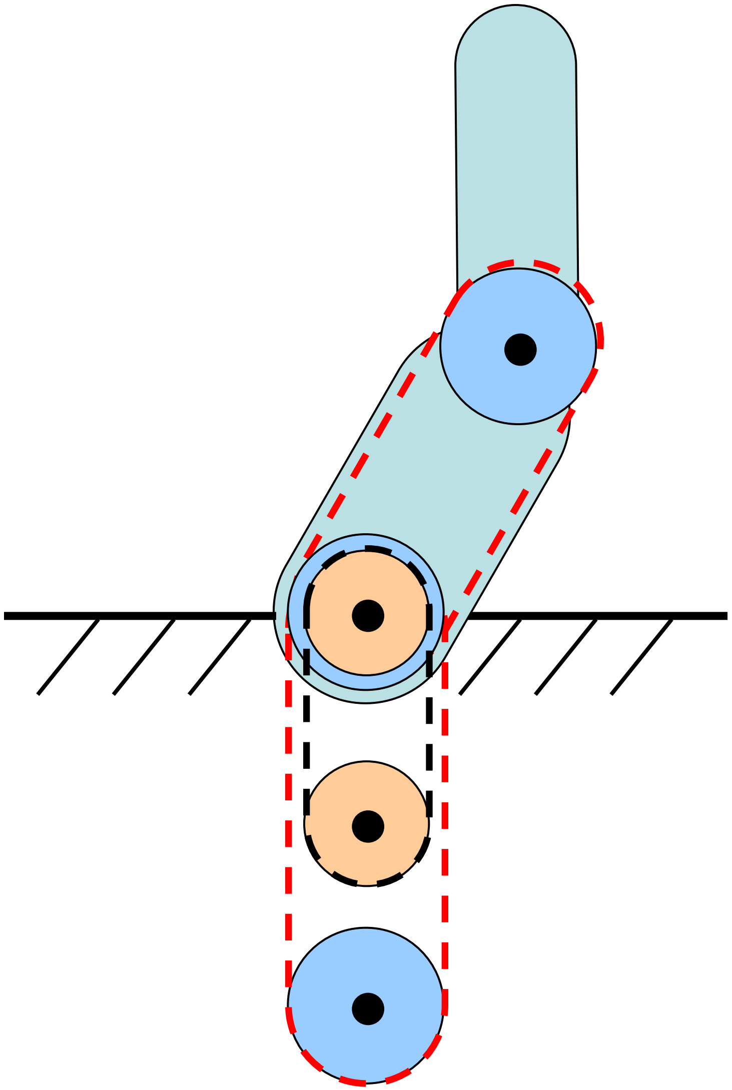
ワイヤプーリ機構の場合



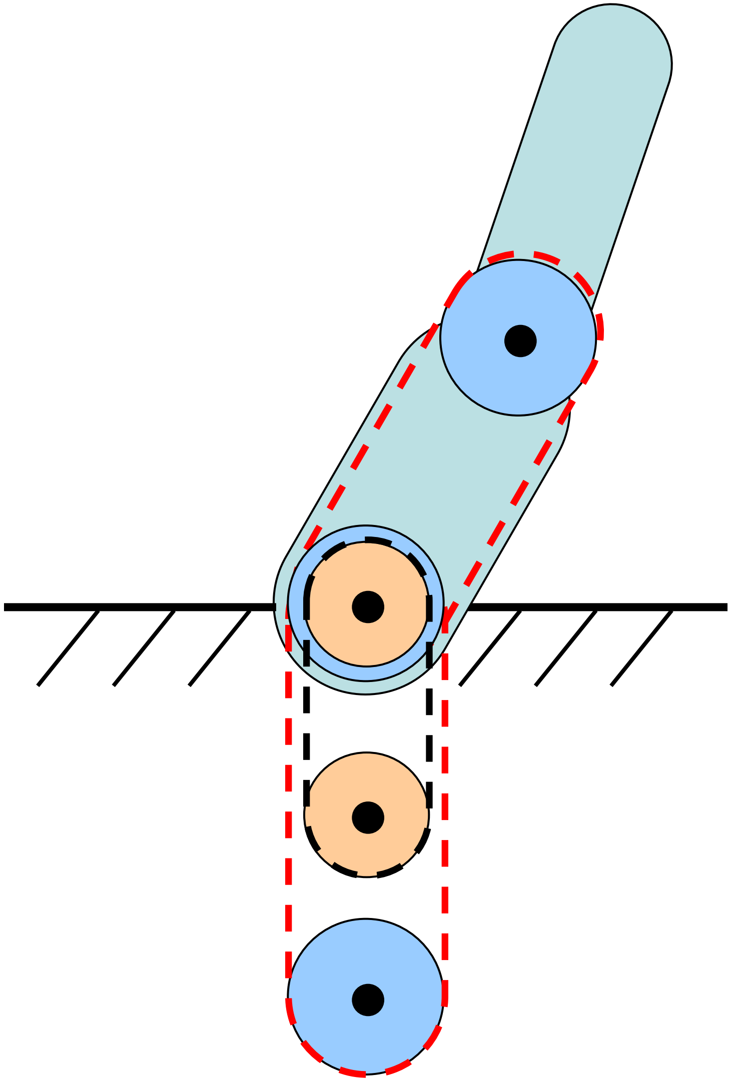
絶対系にモータがついている場合の角度の取り方



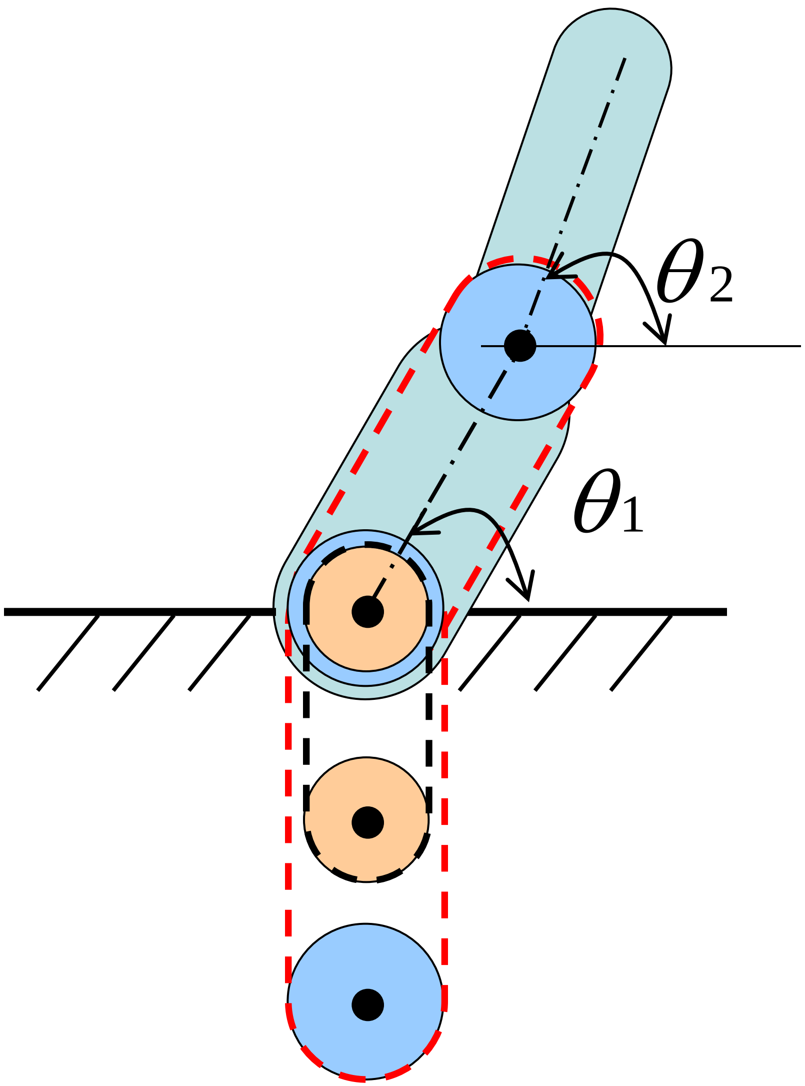
絶対系にモータがついている場合の角度の取り方



絶対系にモータがついている場合の角度の取り方

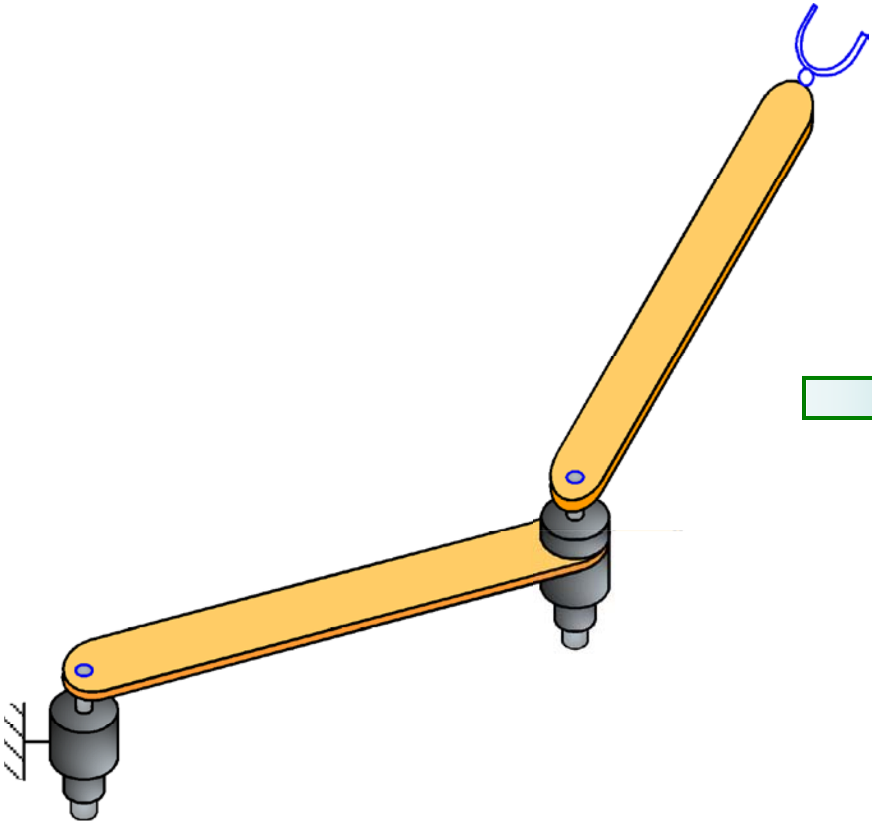


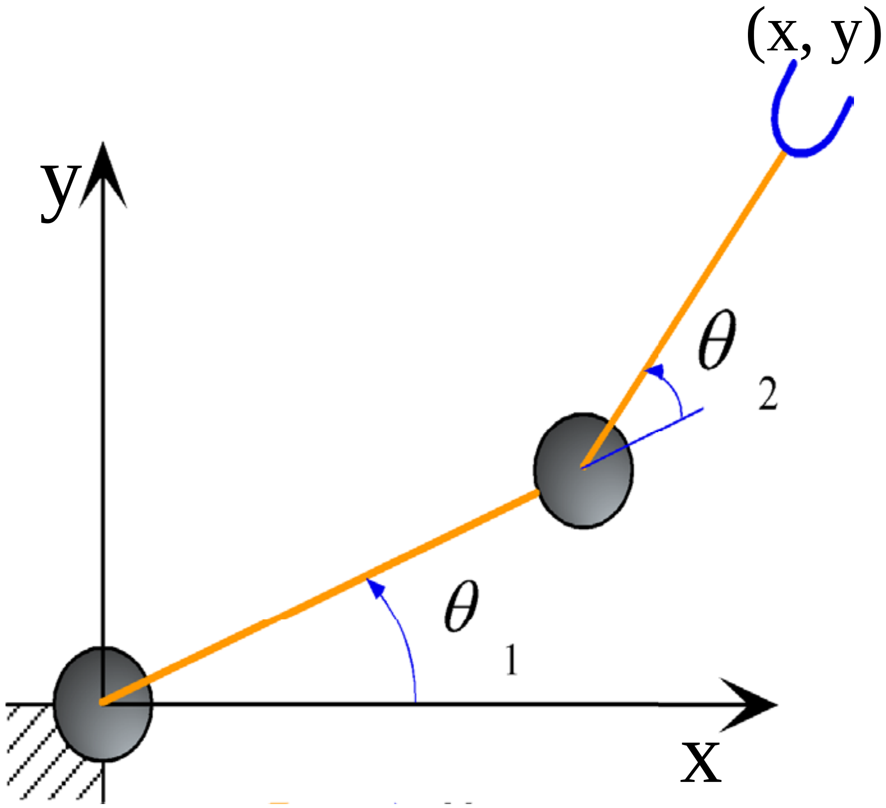
絶対系にモータがついている場合の角度の取り方



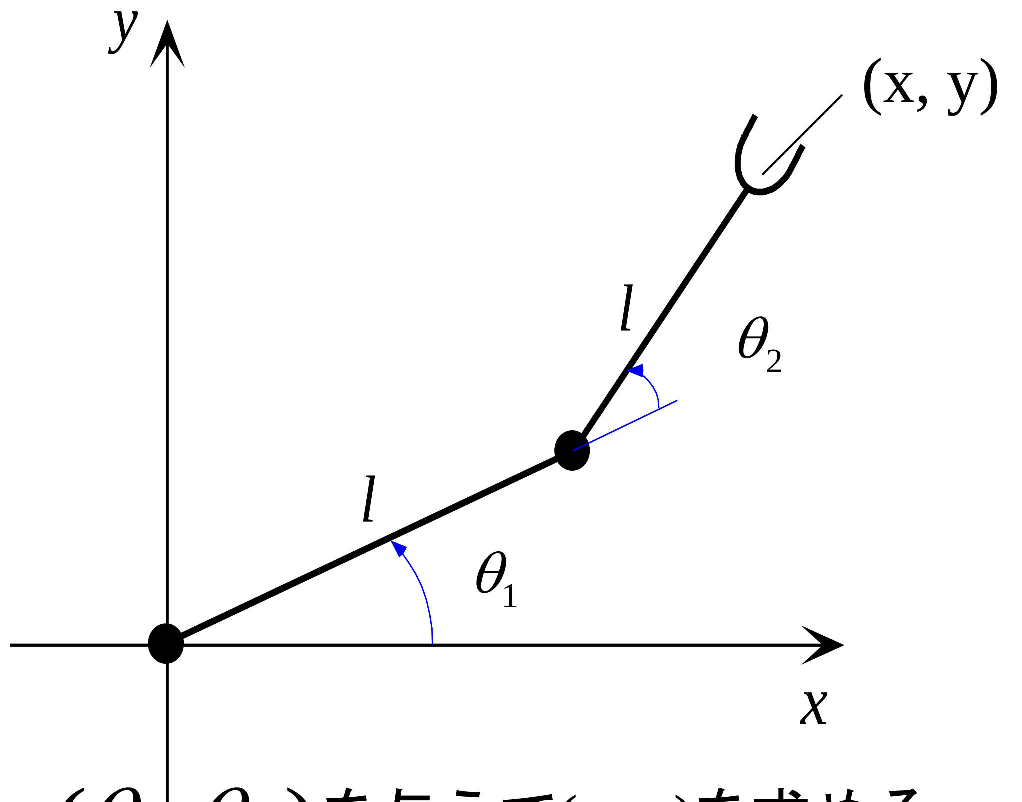
順問題

関節角度を与えて手先位置を求める.





順問題(簡単)

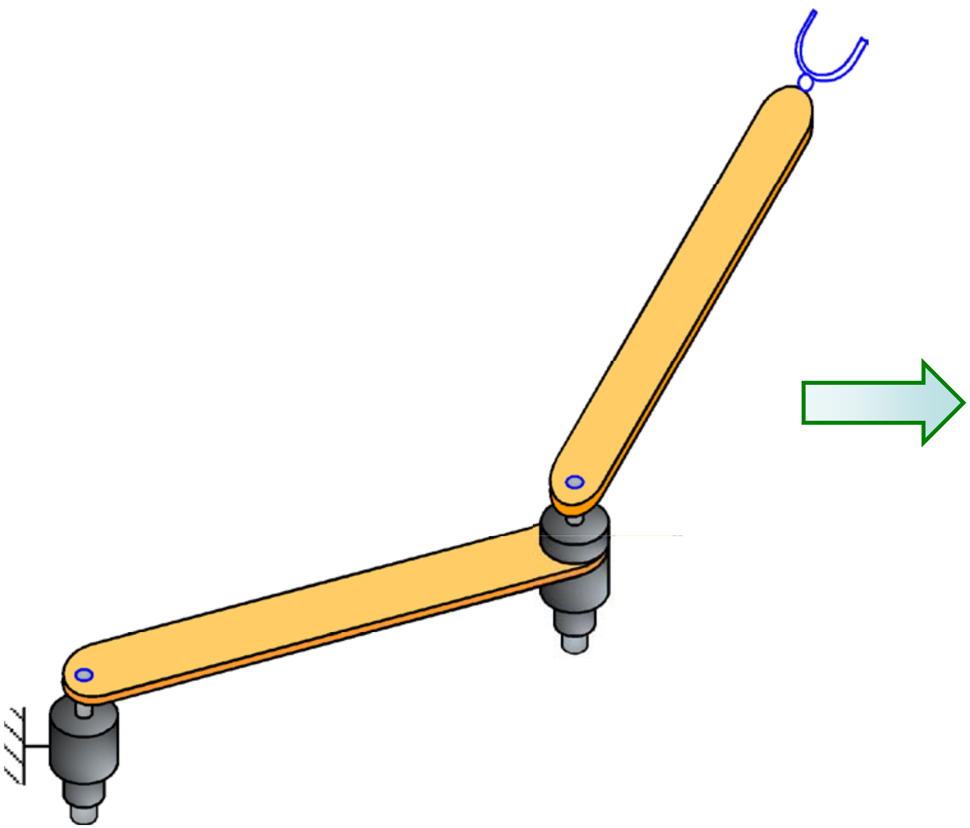


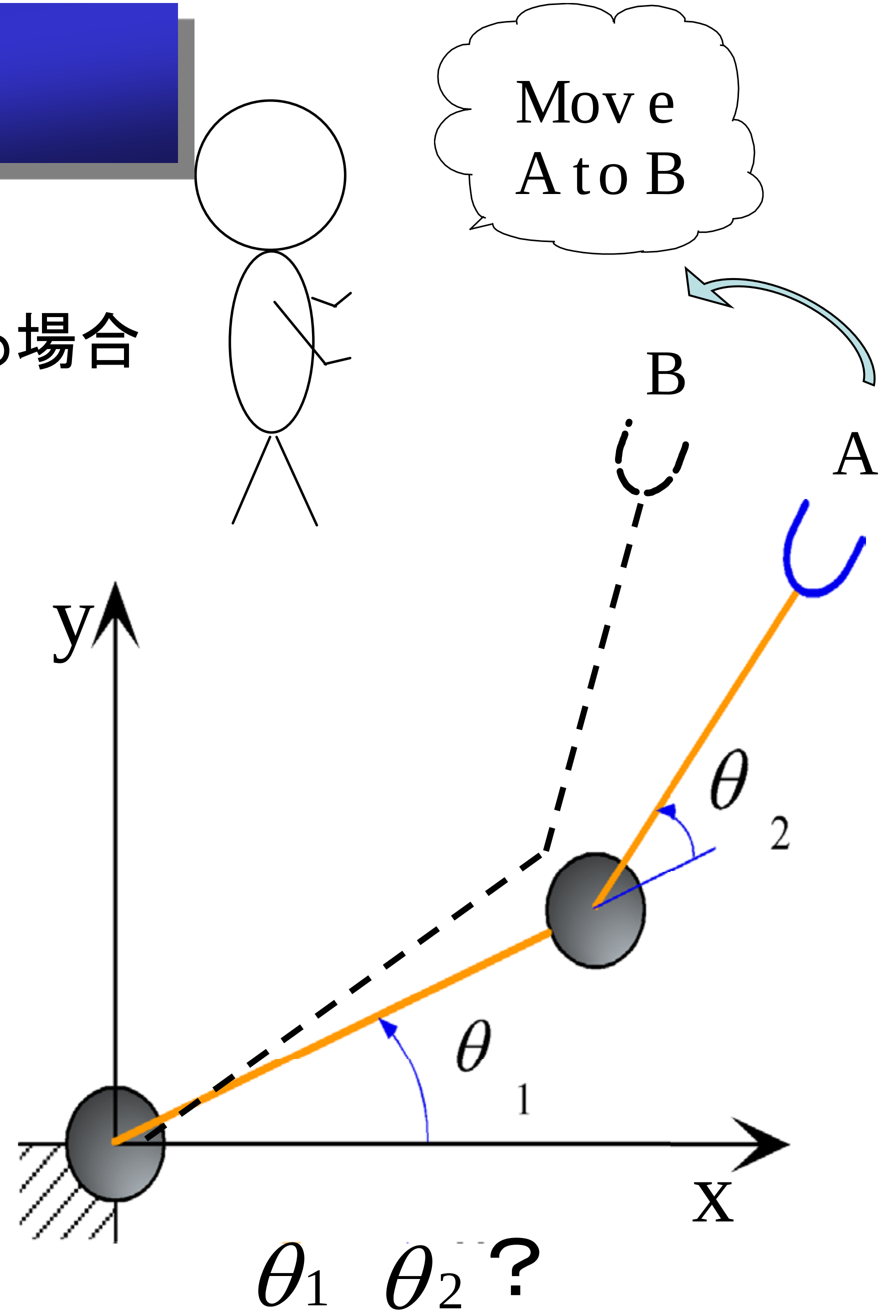
(θ_1, θ_2)を与えて(x, y)を求める

$$\begin{cases} x = l\cos\theta_1 + l\cos(\theta_1+\theta_2) \\ y = l\sin\theta_1 + l\sin(\theta_1+\theta_2) \end{cases}$$

逆問題の必要性

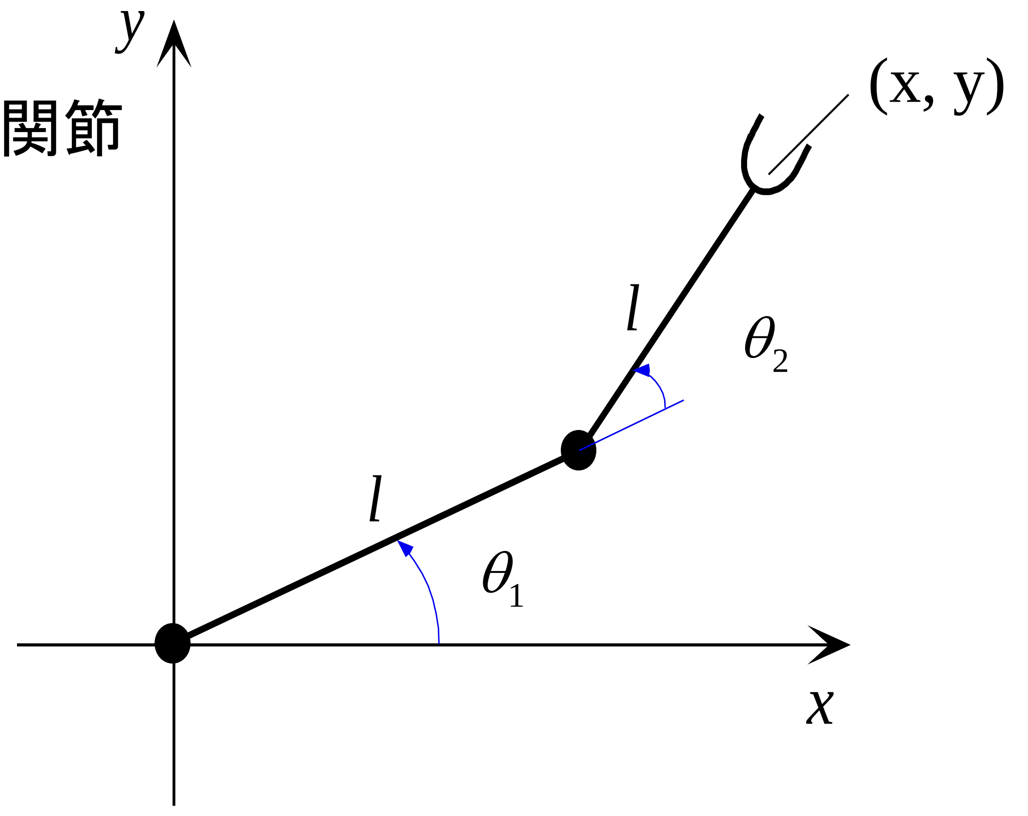
モータが関節に埋め込まれている場合





逆問題

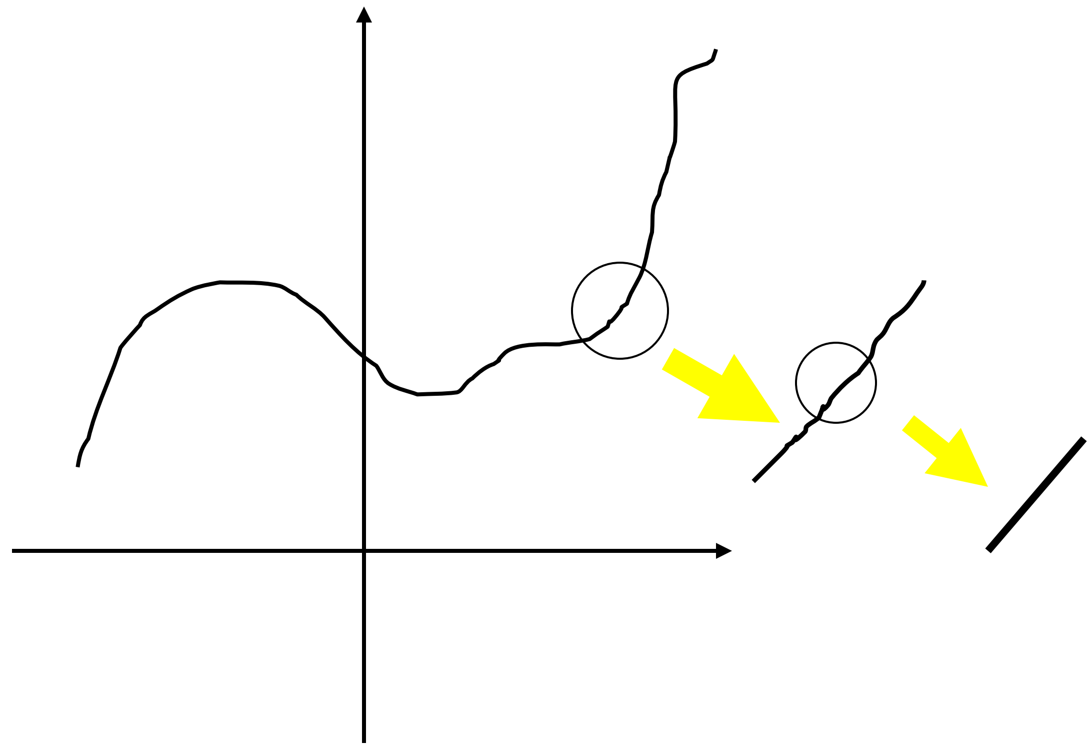
手先位置を与えて関節角度を求める.

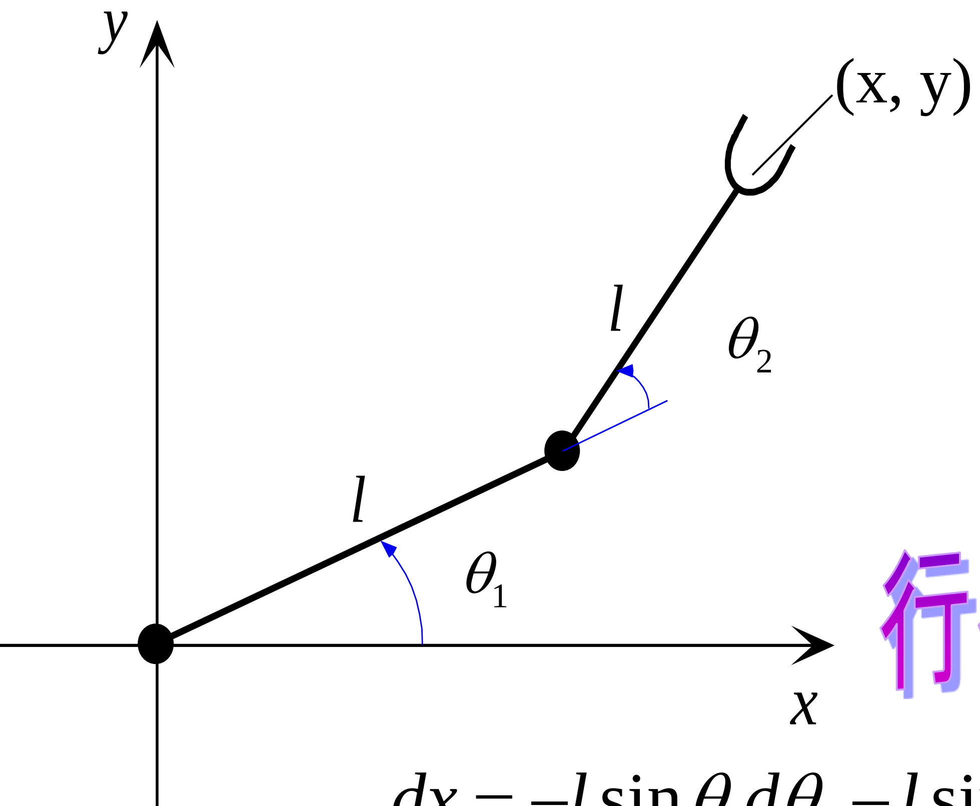


$$\begin{cases} x = l\cos\theta_1 + l\cos(\theta_1+\theta_2) \\ y = l\sin\theta_1 + l\sin(\theta_1+\theta_2) \end{cases}$$

(θ_1, θ_2) ?

非線形関数の線形化





$$\begin{cases} x = l\cos\theta_1 + l\cos(\theta_1+\theta_2) \\ y = l\sin\theta_1 + l\sin(\theta_1+\theta_2) \end{cases}$$

行列表記→線形

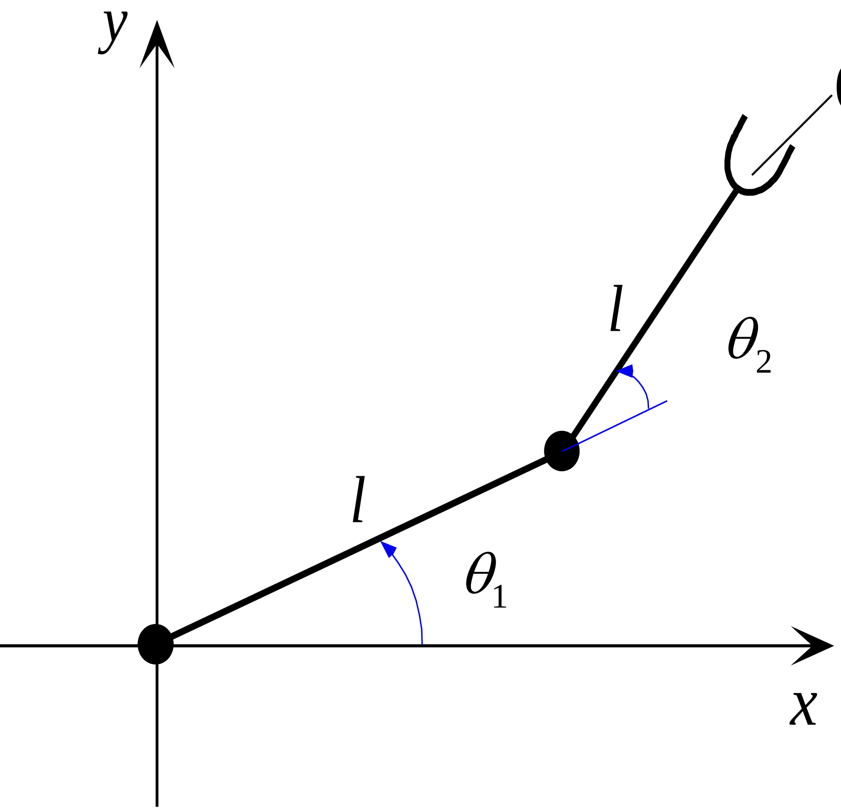
$$dx = -l\sin\theta_1 d\theta_1 - l\sin(\theta_1+\theta_2)(d\theta_1+d\theta_2)$$

$$dy = l\cos\theta_1 d\theta_1 + l(\theta_1+\theta_2)(d\theta_1+d\theta_2)$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = l \underbrace{\begin{pmatrix} -\sin\theta_1 - \sin(\theta_1+\theta_2) & -\sin(\theta_1+\theta_2) \\ \cos\theta_1 + \cos(\theta_1+\theta_2) & \cos(\theta_1+\theta_2) \end{pmatrix}}_{J} \begin{pmatrix} d\theta_1 \\ d\theta_2 \end{pmatrix}$$

$$\mathbf{dx} = \mathrm{J}d\theta$$

J（ヤコビ行列）



$$d\mathbf{x} = \mathrm{J}\,d\theta$$

$$d\theta = \mathrm{J}^{-1} d\mathbf{x} \quad (if\ |\mathrm{J}| \neq 0)$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} -ls_1 - ls_{12} & -ls_{12} \\ lc_1 + lc_{12} & lc_{12} \end{pmatrix} \begin{pmatrix} d\theta_1 \\ d\theta_2 \end{pmatrix}$$

$$c_1 = \cos\theta_1, \quad s_1 = \sin\theta_1$$

$$c_{12} = \cos(\theta_1 + \theta_2), \quad s_{12} = \sin(\theta_1 + \theta_2)$$

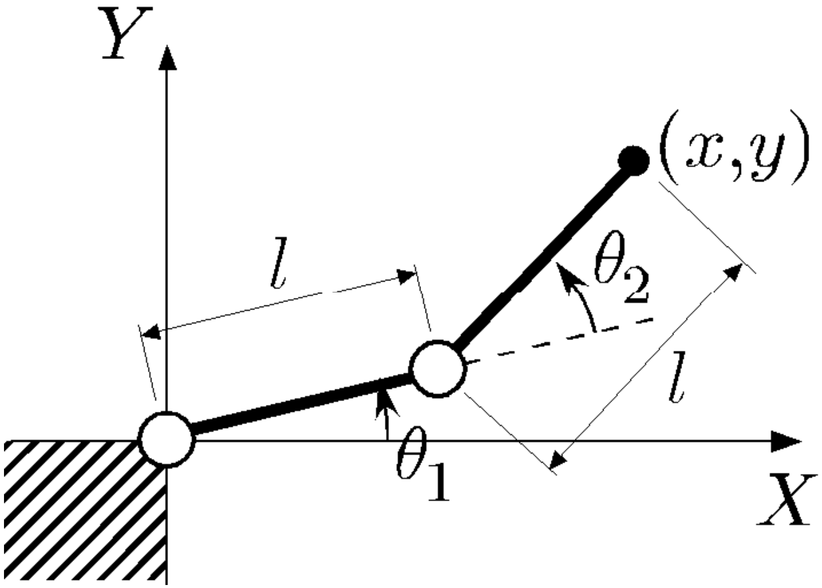
$$\mathrm{J}^{-1} = \frac{1}{|\mathrm{J}|} \begin{pmatrix} lc_{12} & ls_{12} \\ -lc_1 - lc_{12} & -ls_1 - ls_{12} \end{pmatrix}$$

$$|\mathrm{J}| = l^2 \{- s_1 c_{12} - s_{12} c_{12} + s_{12} c_1 + s_{12} c_{12}\}$$
$$= l^2 \sin\theta_2$$

特異姿勢：2リンクロボットの場合

$$d\mathbf{x} = \boldsymbol{J}\, d\boldsymbol{\theta}$$

$$dx = -lS_1 d\theta_1 - lS_{12}(d\theta_1 + d\theta_2)$$
$$dy = lC_1 d\theta_1 + lC_{12}(d\theta_1 + d\theta_2)$$

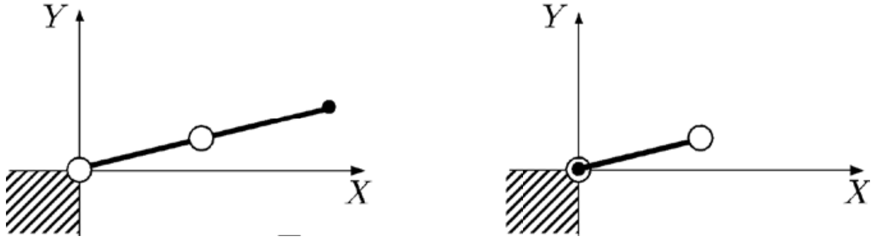


$$\therefore \boldsymbol{J} = \begin{bmatrix} -lS_1 - lS_{12} & -lS_{12} \\ lC_1 + lC_{12} & lC_{12} \end{bmatrix}$$

$$S_1 = \sin\theta_1, \quad S_{12} = \sin(\theta_1 + \theta_2)$$
$$C_1 = \cos\theta_1, \quad C_{12} = \cos(\theta_1 + \theta_2)$$

$$|\mathrm{J}| = 0$$

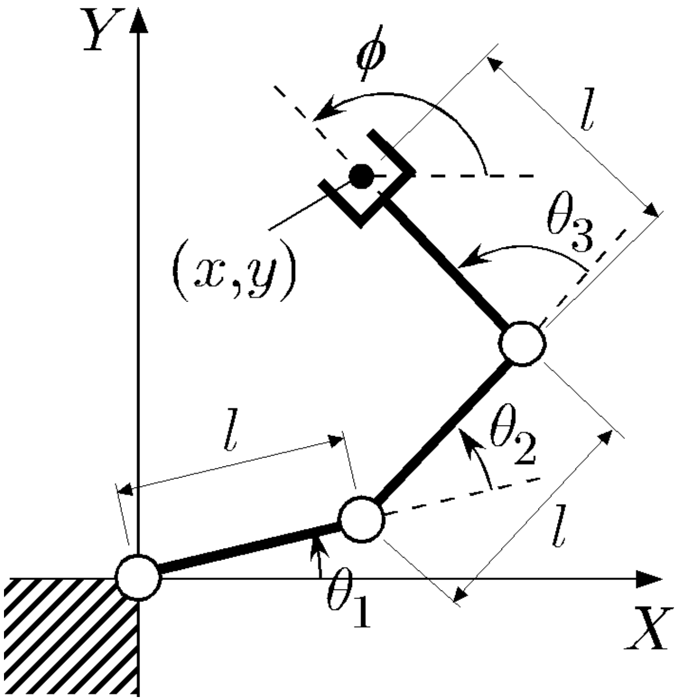
特異姿勢での作業はヒトでもきつい



$$l^2 \sin\theta_2 = 0 \quad \therefore \theta_2 = n\pi$$

課題1：3リンクロボットについて以下の問いに答えよ

手先位置・姿勢(x, y, φ)に対して特異姿勢を求め図示せよ.



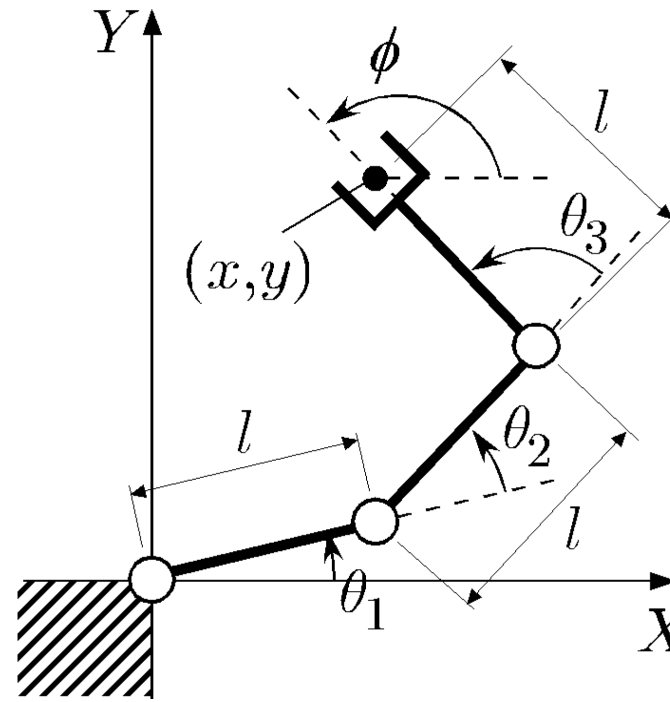
課題1：　解答

$$dx = J\, d\theta$$

$$dx = -lS_1 d\theta_1 - lS_{12}(d\theta_1 + d\theta_2) - lS_{123}(d\theta_1 + d\theta_2 + d\theta_3)$$

$$dy = lC_1 d\theta_1 + lC_{12}(d\theta_1 + d\theta_2) + lC_{123}(d\theta_1 + d\theta_2 + d\theta_3)$$

$$d\phi = d\theta_1 + d\theta_2 + d\theta_3$$



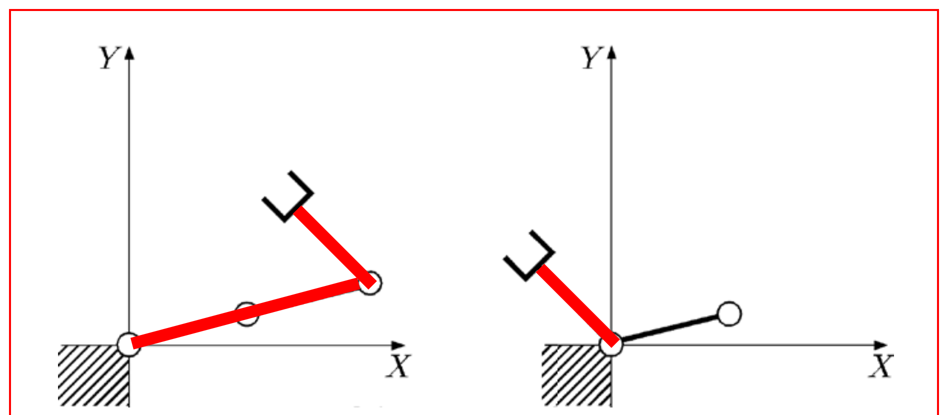
$$\therefore J = \begin{bmatrix} -lS_1 - lS_{12} - lS_{123} & -lS_{12} - lS_{123} & -lS_{123} \\ lC_1 + lC_{12} + lC_{123} & lC_{12} + lC_{123} & lC_{123} \\ 1 & 1 & 1 \end{bmatrix}$$

$S_1 = \sin\theta_1,\quad S_{12} = \sin(\theta_1 + \theta_2),\quad S_{123} = \sin(\theta_1 + \theta_2 + \theta_3)$

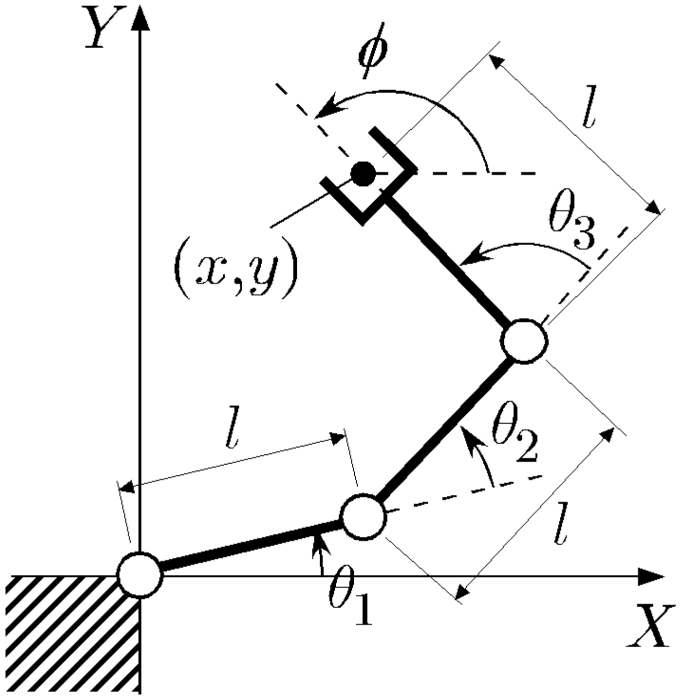
$C_1 = \cos\theta_1,\quad C_{12} = \cos(\theta_1 + \theta_2),\quad C_{123} = \cos(\theta_1 + \theta_2 + \theta_3)$

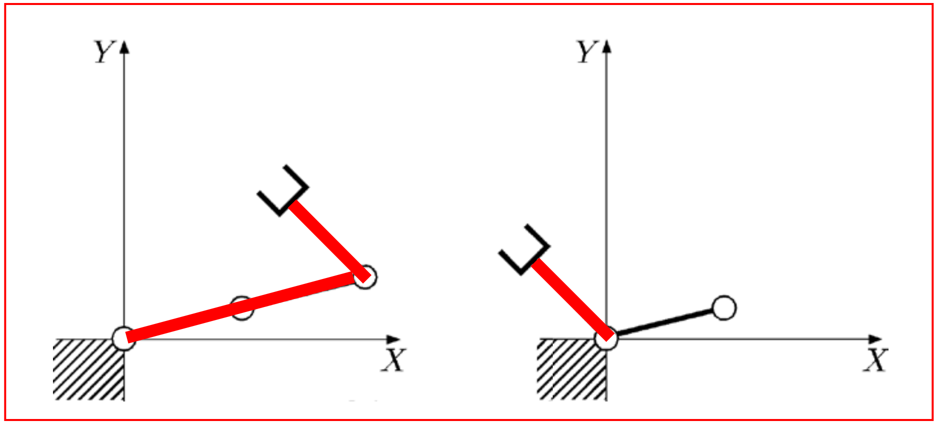
$$|J| = 0$$

$$l^2 \sin\theta_2 = 0 \qquad \therefore \theta_2 = n\pi$$

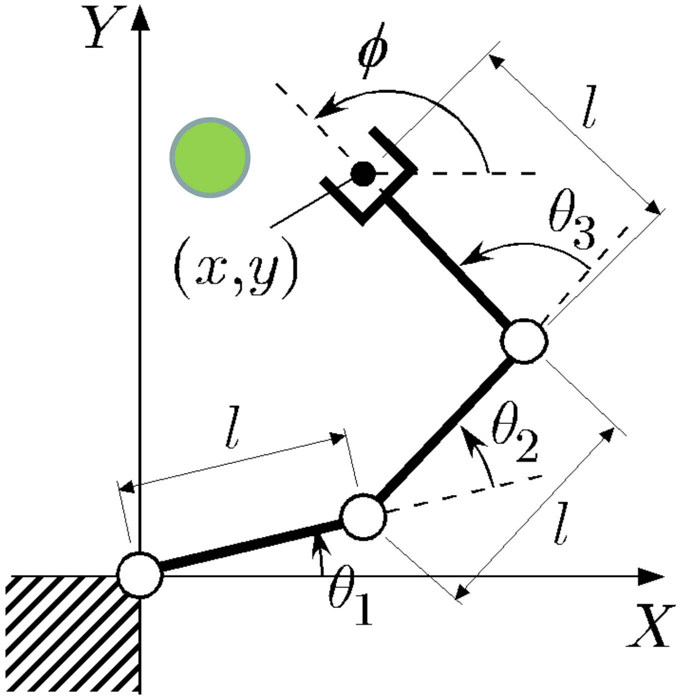


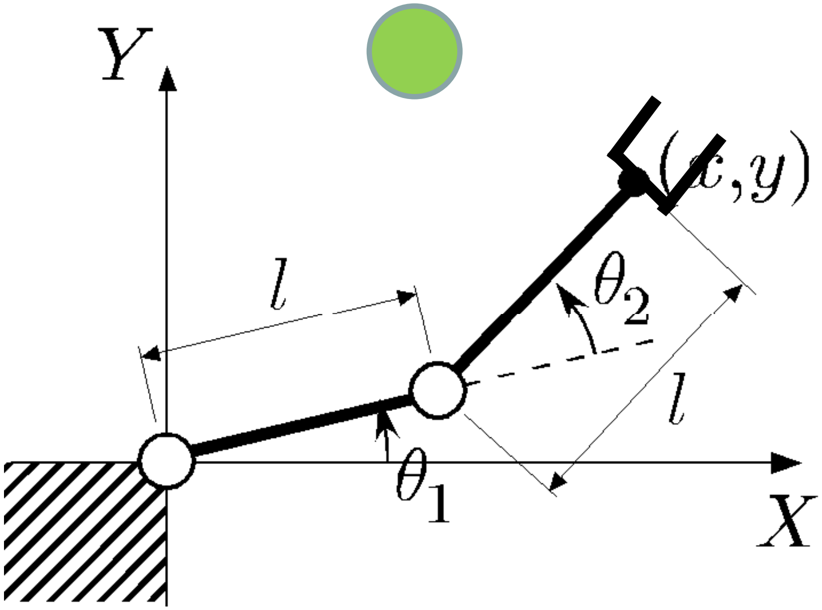
特異姿勢が存在しない2次元3自由度ロボットは存在するか？



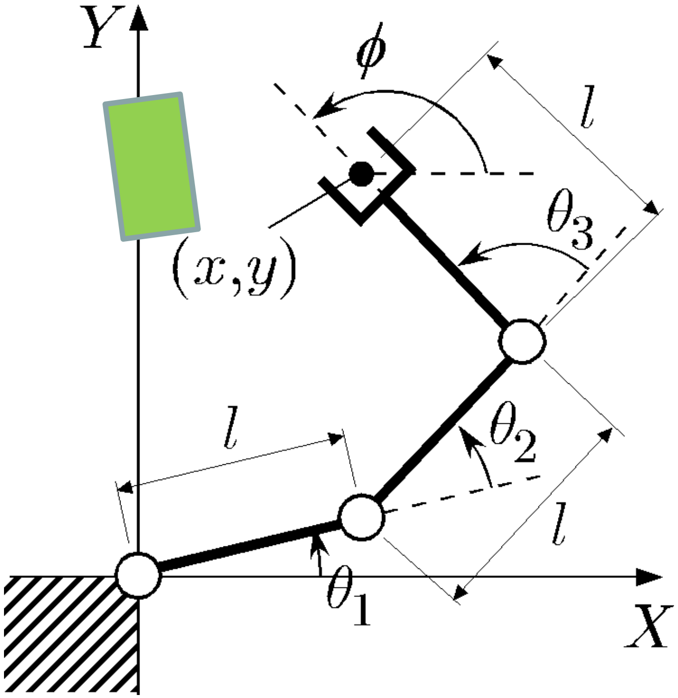


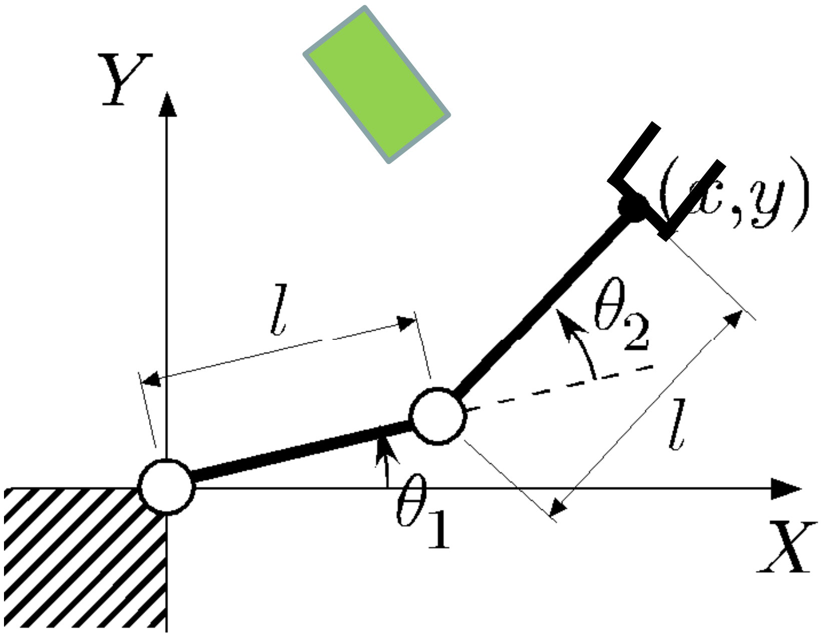
３リンクロボットの２リンクロボットに対する運動学的優位性は？



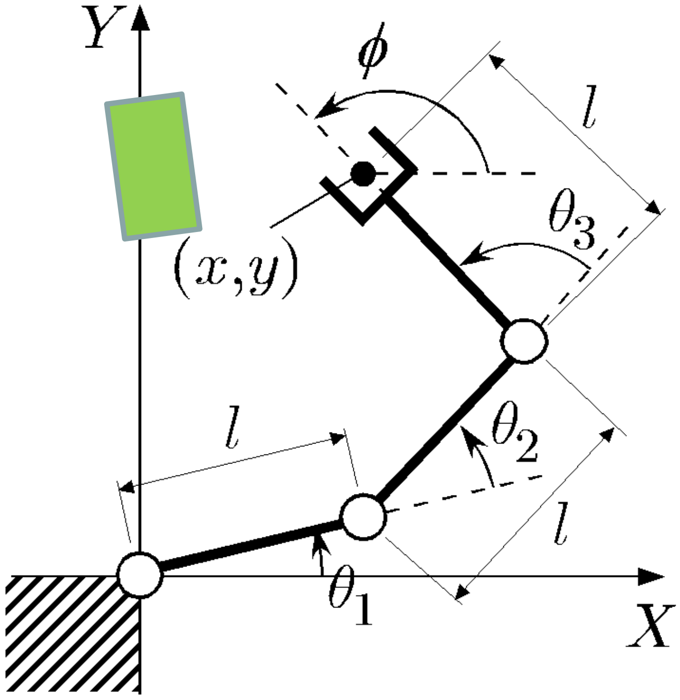


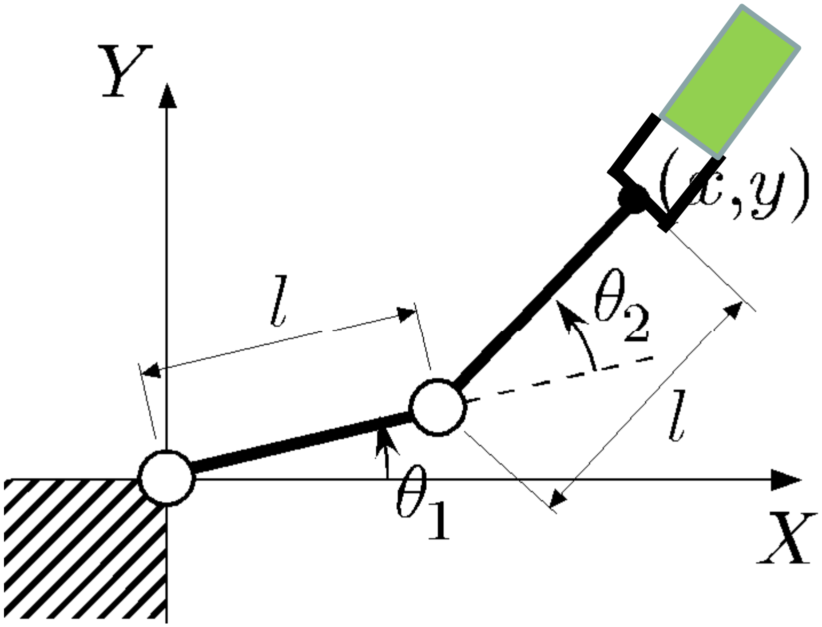
３リンクロボットの２リンクロボットに対する運動学的優位性は？



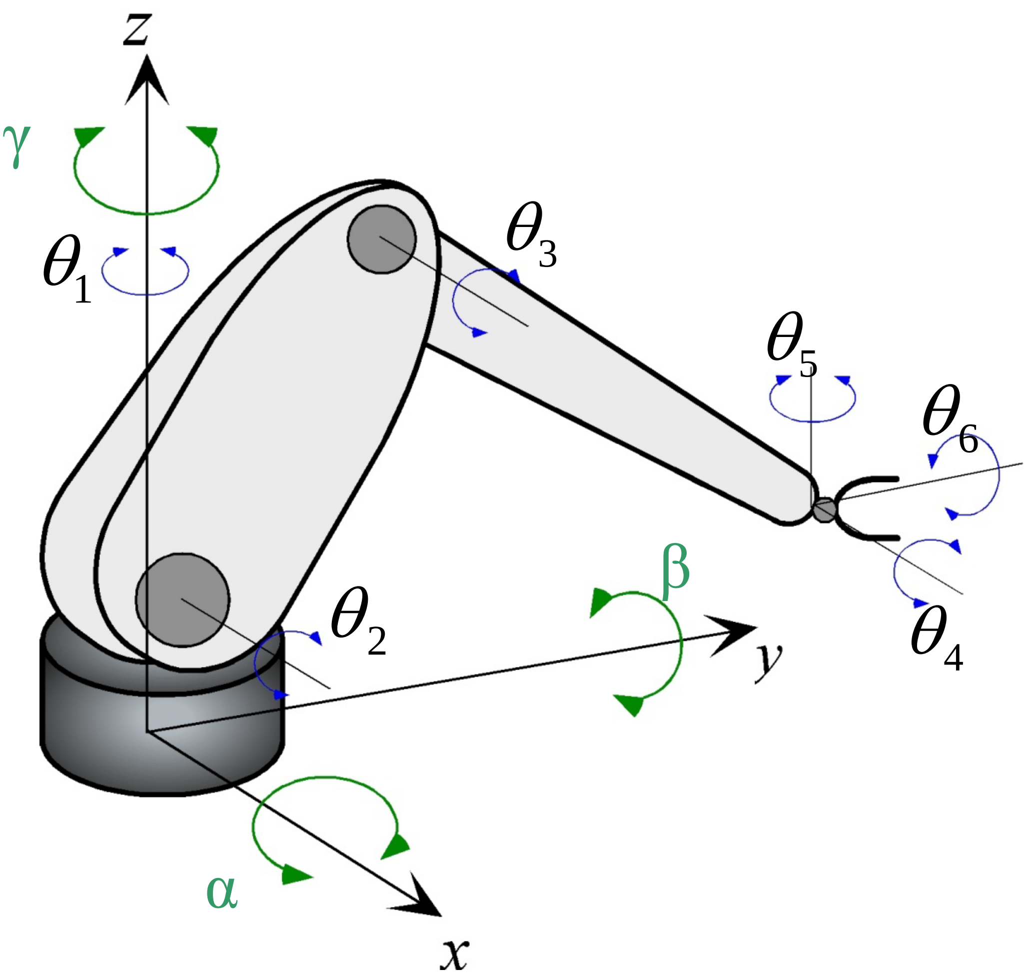


3リンクロボットの２リンクロボットに対する運動学的優位性は？





６自由度ロボットの場合



$$x = f_1(\theta_1, \theta_2, \cdots, \theta_6)$$

$$y = f_2(\theta_1, \theta_2, \cdots, \theta_6)$$

$$z = f_3(\theta_1, \theta_2, \cdots, \theta_6)$$

$$\alpha = f_4(\theta_1, \theta_2, \cdots, \theta_6)$$

$$\beta = f_5(\theta_1, \theta_2, \cdots, \theta_6)$$

$$\gamma = f_6(\theta_1, \theta_2, \cdots, \theta_6)$$

$$\Delta \mathbf{x} = \mathrm{J} \Delta \theta$$

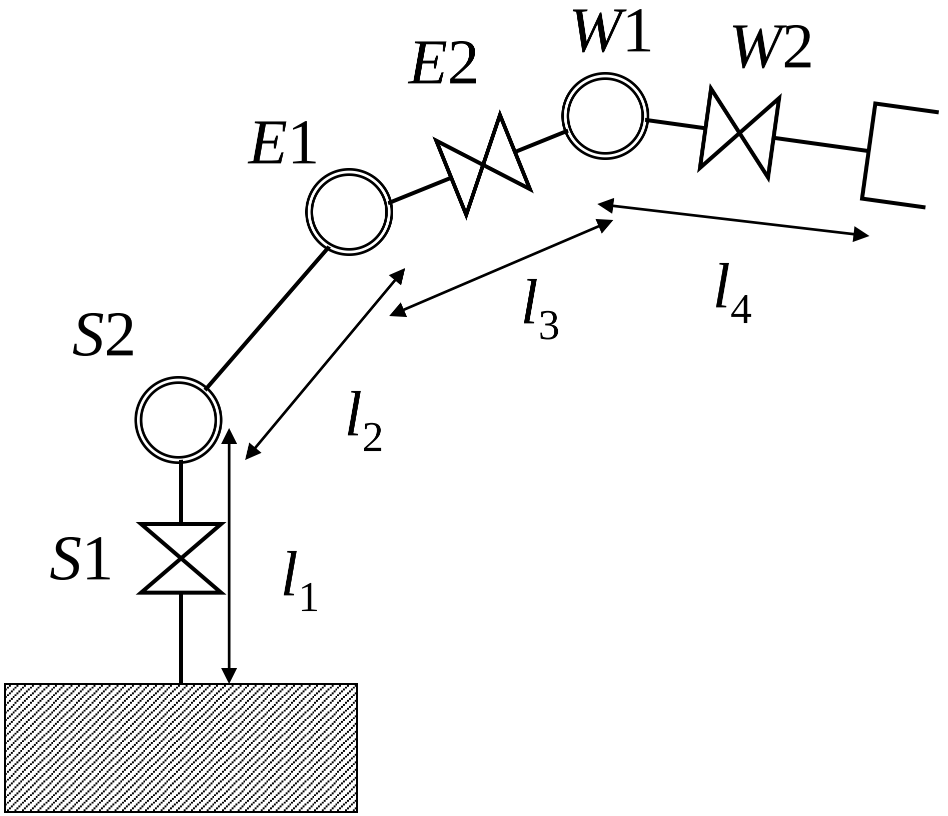
$$\mathrm{J} = \begin{pmatrix} \frac{\partial f_1}{\partial \theta_1} & \cdots & \frac{\partial f_1}{\partial \theta_6} \\ \vdots & & \vdots \\ \frac{\partial f_6}{\partial \theta_1} & \cdots & \frac{\partial f_6}{\partial \theta_6} \end{pmatrix}$$

６自由度ロボットの特異姿勢

$$d\mathbf{x} = \boldsymbol{J} d\boldsymbol{\theta} \Longleftrightarrow d\boldsymbol{\theta} = \boldsymbol{J}^{-1} d\mathbf{x}$$

$$\boldsymbol{J} \in R^{6\times6}$$

$$d\mathbf{x} = \begin{bmatrix} x \\ y \\ z \\ \theta_x \\ \theta_y \\ \theta_z \end{bmatrix} \quad d\boldsymbol{\theta} = \begin{bmatrix} S1 \\ S2 \\ E1 \\ E2 \\ W1 \\ W2 \end{bmatrix}$$



$$\det \boldsymbol{J} = 0$$

$$-l_2 l_3 \mathrm{Sin}[E1]\,\mathrm{Sin}[W1](l_2 \mathrm{Sin}[S2] + l_3 \mathrm{Sin}[E1+S2]) = 0$$

６自由度ロボットの特異姿勢

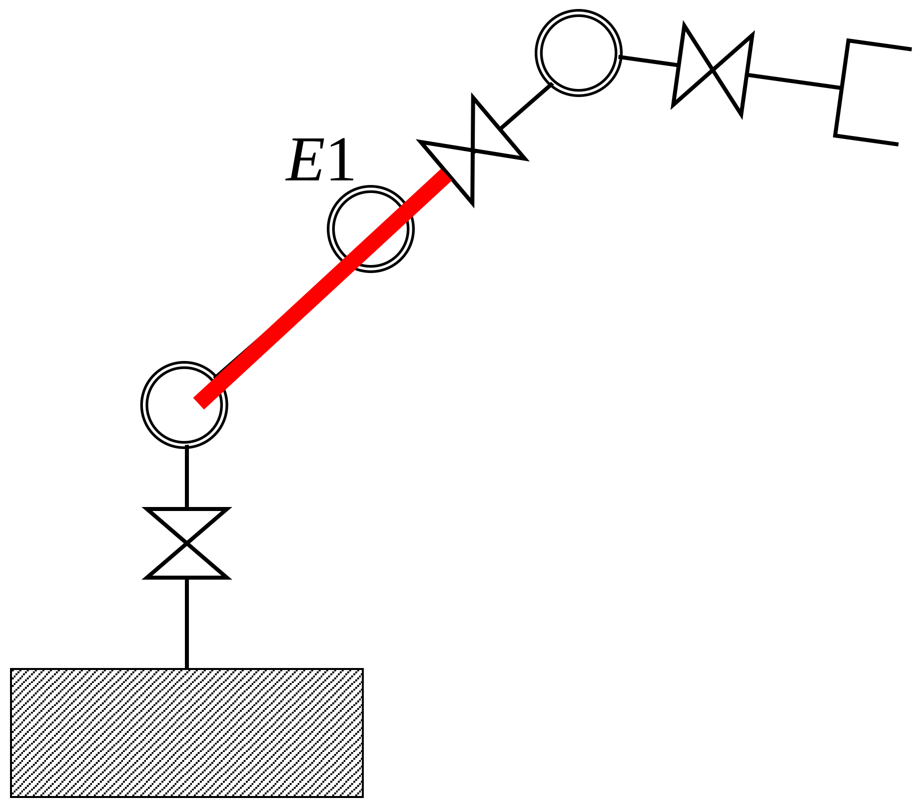
$$-l_2 l_3 \underbrace{\mathrm{Sin}[E1]}_{①}\ \underbrace{\mathrm{Sin}[W1]}_{②}\underbrace{(l_2 \mathrm{Sin}[S2] + l_3 \mathrm{Sin}[E1+S2])}_{③} = 0$$

①肘特異姿勢

$$\mathrm{Sin}[E1] = 0$$

$$E1 = n\pi$$

（n :整数）



６自由度ロボットの特異姿勢

$$-l_2 l_3 \underbrace{\mathrm{Sin}[E1]}_{①} \underbrace{\mathrm{Sin}[W1]}_{②} \underbrace{(l_2 \mathrm{Sin}[S2] + l_3 \mathrm{Sin}[E1+S2])}_{③} = 0$$

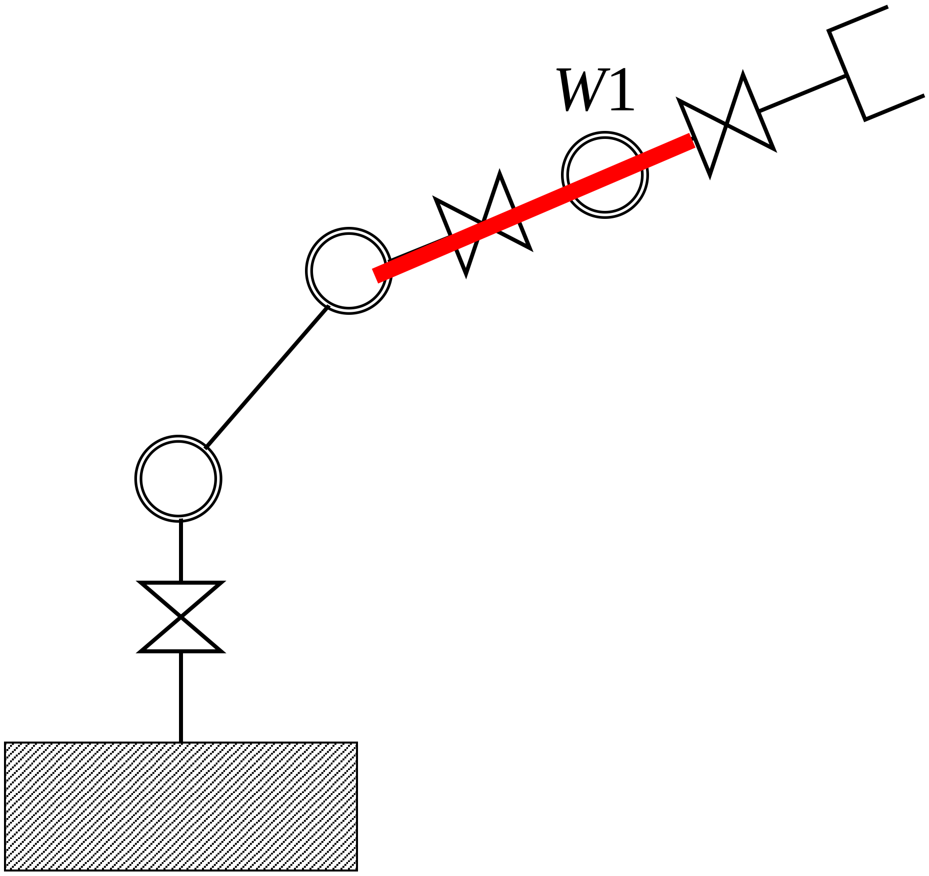
②手首特異姿勢

$$\mathrm{Sin}[W1] = 0$$

⇩

$$W1 = n\pi$$

（n :整数）

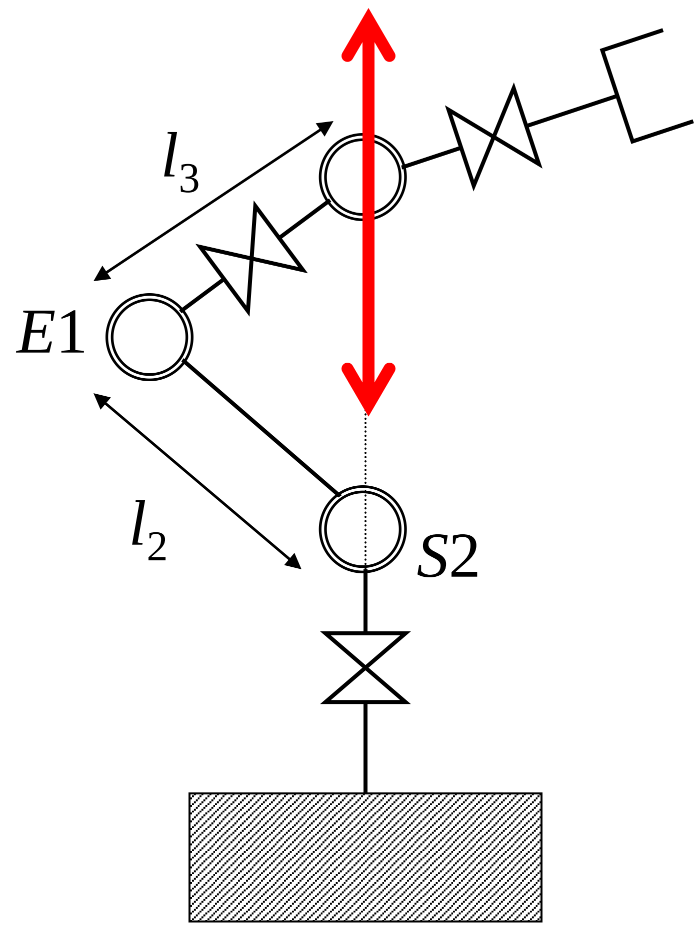


６自由度ロボットの特異姿勢

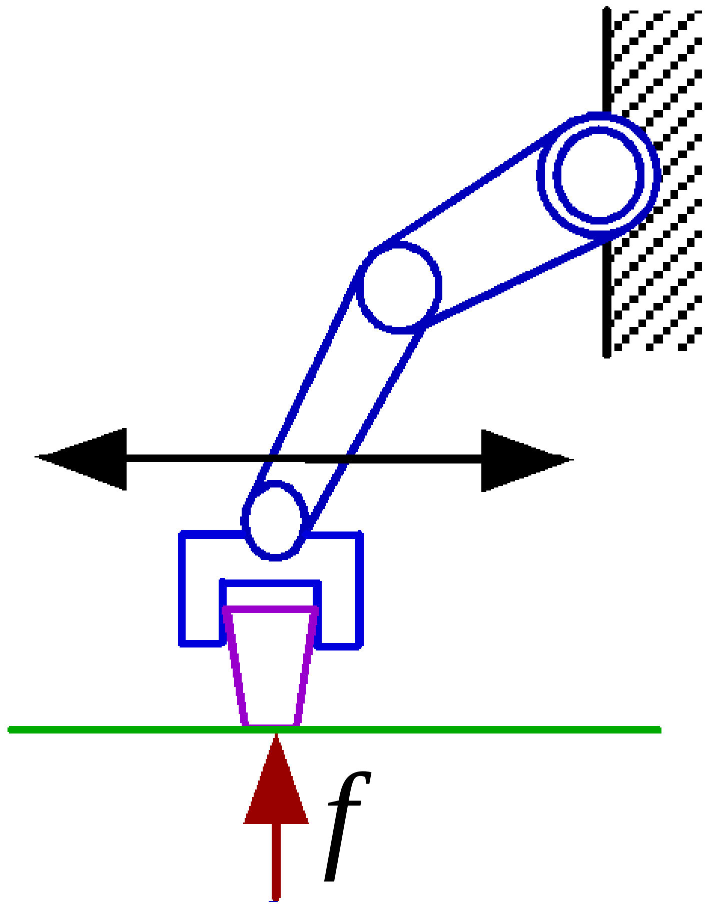
$$-l_2 l_3 \underset{①}{\underline{\mathrm{Sin}[E1]}}\ \underset{②}{\underline{\mathrm{Sin}[W1]}}\ \underset{③}{\underline{(l_2 \mathrm{Sin}[S2] + l_3 \mathrm{Sin}[E1+S2])}} = 0$$

③肩特異姿勢

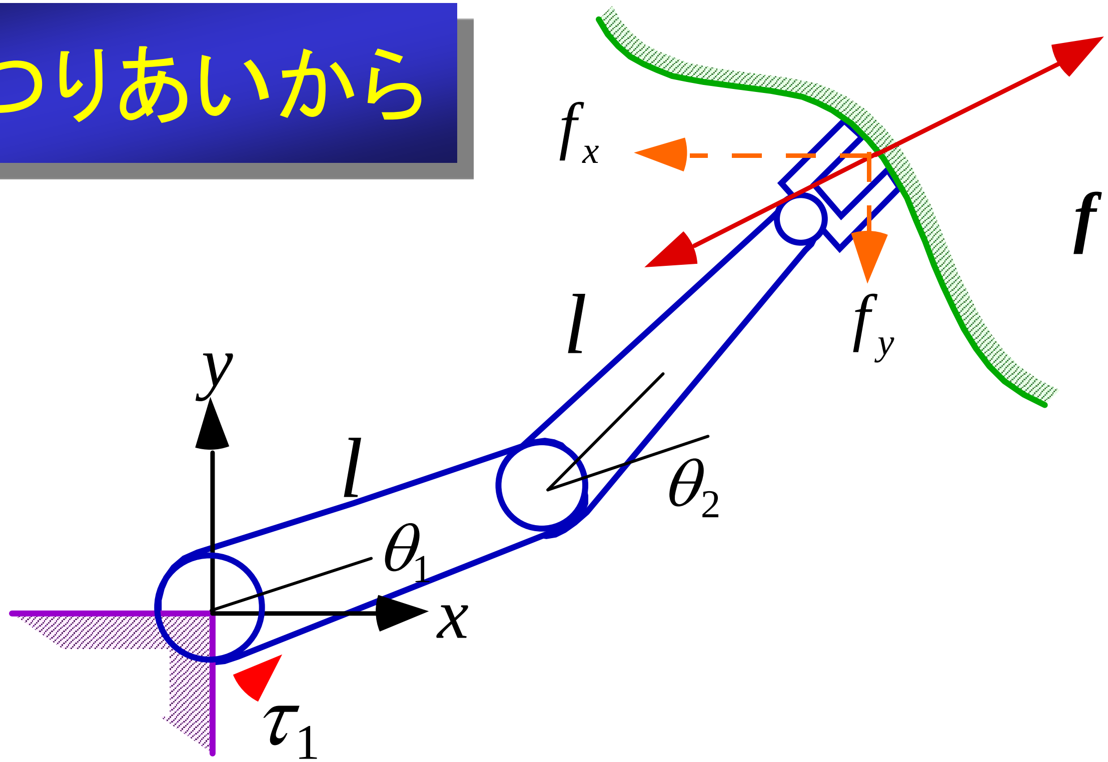
$$l_2 \mathrm{Sin}[S2] + l_3 \mathrm{Sin}[E1+S2] = 0$$



ロボットで力を環境に加えるには？



力のつりあいから



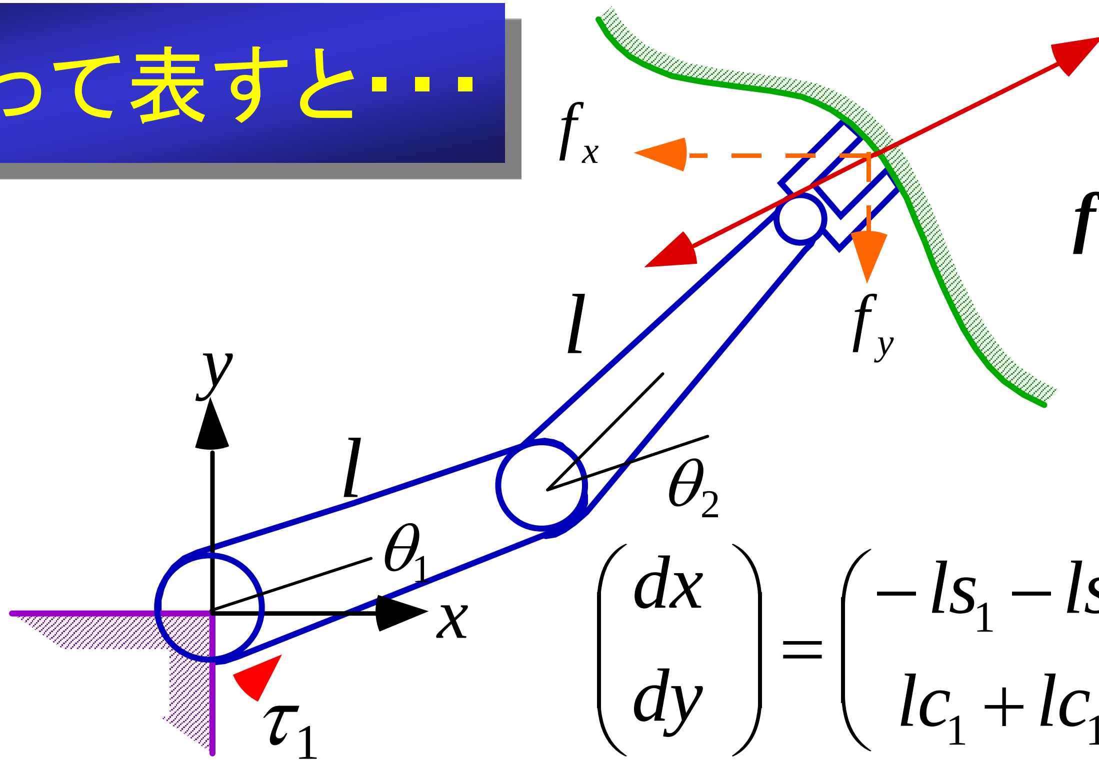
$$\boldsymbol{f} = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

$$\begin{cases} \tau_1 - f_y\{l\cos\theta_1 + l\cos(\theta_1+\theta_2)\} + f_x\{l\sin\theta_1 + l\sin(\theta_1+\theta_2)\} = 0 \\ \tau_2 - f_y l\cos(\theta_1+\theta_2) + f_x l\sin(\theta_1+\theta_2) = 0 \end{cases}$$

$$\Rightarrow \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} -ls_1 - ls_{12} & lc_1 + lc_{12} \\ -ls_{12} & lc_{12} \end{pmatrix} \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

$s_1 = \sin\theta_1 \quad s_{12} = \sin(\theta_1+\theta_2) \quad c_1 = \cos\theta_1 \quad c_{12} = \cos(\theta_1+\theta_2)$

Jを使って表すと・・・



$$\mathbf{f} = \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} -ls_1 - ls_{12} & -ls_{12} \\ lc_1 + lc_{12} & lc_{12} \end{pmatrix} \begin{pmatrix} d\theta_1 \\ d\theta_2 \end{pmatrix}$$

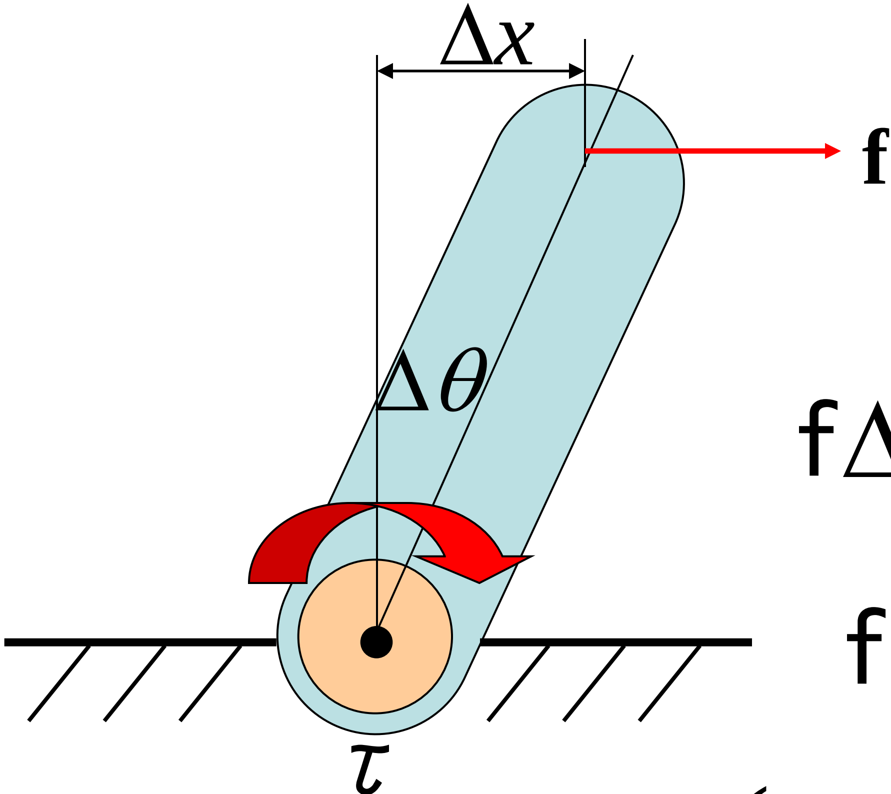
$$d\boldsymbol{x} = J d\boldsymbol{\theta}$$

$$\begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix} = \begin{pmatrix} -ls_1 - ls_{12} & lc_1 + lc_{12} \\ -ls_{12} & lc_{12} \end{pmatrix} \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

$$\boldsymbol{\tau} = J^t \mathbf{f}$$

$$s_1 = \sin\theta_1 \quad s_{12} = \sin(\theta_1 + \theta_2) \quad c_1 = \cos\theta_1 \quad c_{12} = \cos(\theta_1 + \theta_2)$$

仮想仕事の原理



$$f\Delta x = \tau\Delta\theta$$

$$f = \tau / l$$

$$(\because \Delta x = l\Delta\theta)$$

仮想仕事の原理による $\boldsymbol{\tau} = J^t \boldsymbol{f}$ の誘導

(1) 関節アクチュエータがする仕事

$$W_j = \tau_1 d\theta_1 + \tau_2 d\theta_2$$

$$= \begin{pmatrix} d\theta_1 & d\theta_2 \end{pmatrix} \begin{pmatrix} \tau_1 \\ \tau_2 \end{pmatrix}$$

$$= d\boldsymbol{\theta}^t \tau$$

(2) 手先力がする仕事

$$W_e = f_x dx + f_y dy$$

$$= \begin{pmatrix} dx & dy \end{pmatrix} \begin{pmatrix} f_x \\ f_y \end{pmatrix}$$

$$= d\boldsymbol{x}^t \boldsymbol{f}$$

$W_j = W_e$ だから

$$d\boldsymbol{\theta}^t \boldsymbol{\tau} = d\boldsymbol{x}^t \boldsymbol{f}$$

$$= (J d\boldsymbol{\theta})^t \boldsymbol{f}$$

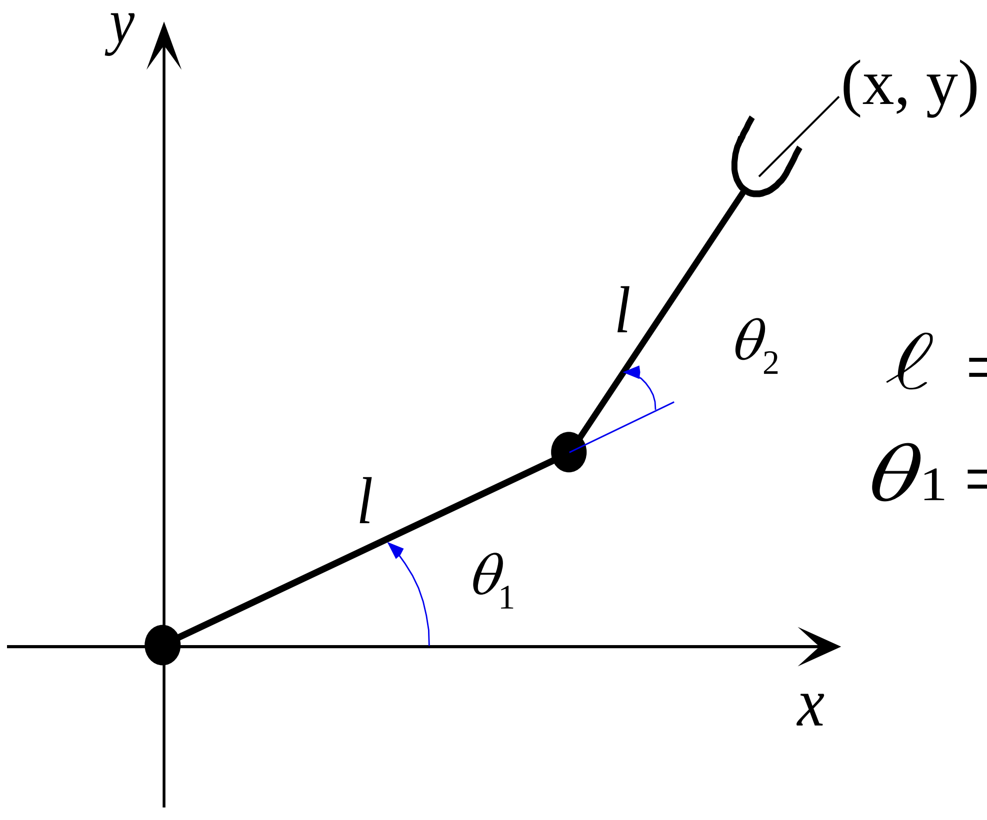
$$= d\boldsymbol{\theta}^t J^t \boldsymbol{f}$$

$$\therefore \boldsymbol{\tau} = J^t \boldsymbol{f}$$

課題：2リンクロボットに対して次の問いに答えよ

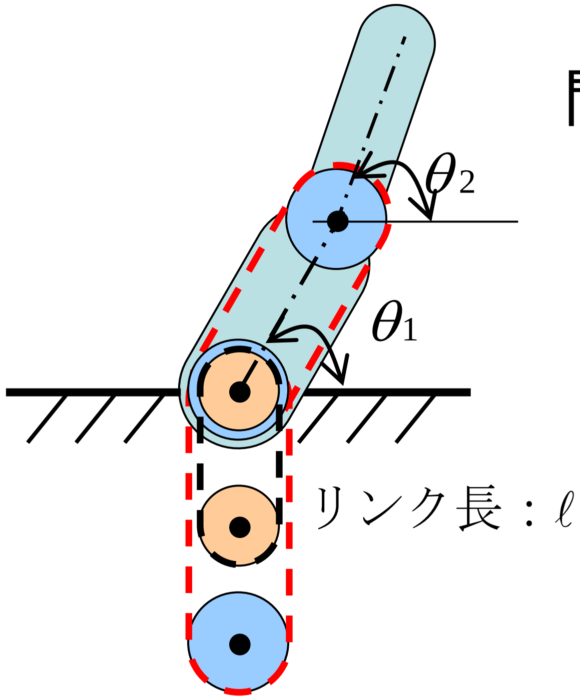
ロボット先端に(fx, fy)=(1.0, 1.0)[N]発生させるためには関節トルク

(τ_1, τ_2) を何[Nm]加えたらよいか.



$$\ell = 1[m]$$

$$\theta_1 = \theta_2 = \pi / 6$$

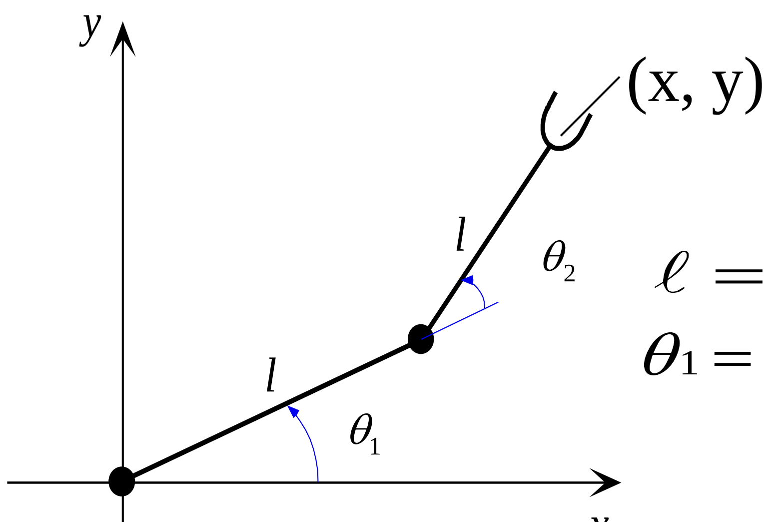


問題1.　(1)ヤコビ行列Jを求めよ.

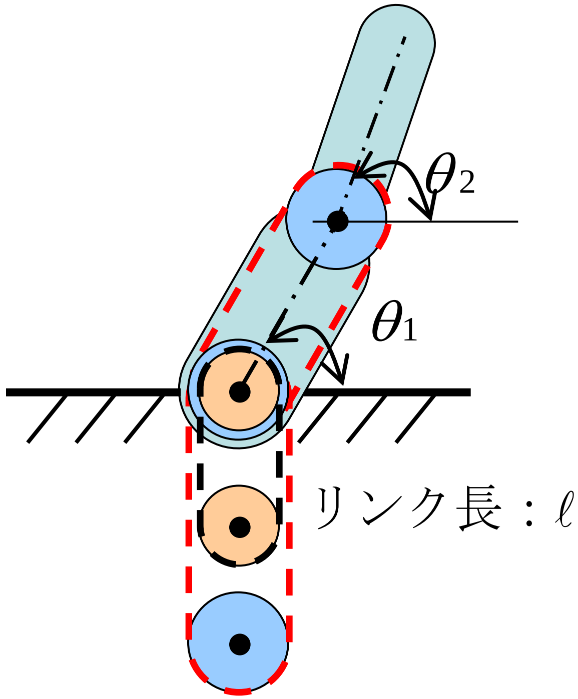
(2) 特異姿勢を与える (θ_1, θ_2) を求めよ.

＊リンクの幅は無視してよい.

問題2　ロボット先端を$(\Delta x, \Delta y)=(0.1, 0.1)$[m]移動させるのに必要な各関節の角度変化 $(\Delta\theta_1, \Delta\theta_2)$ を求めよ.



$$\ell = 1[m]$$
$$\theta_1 = \theta_2 = \pi / 6$$

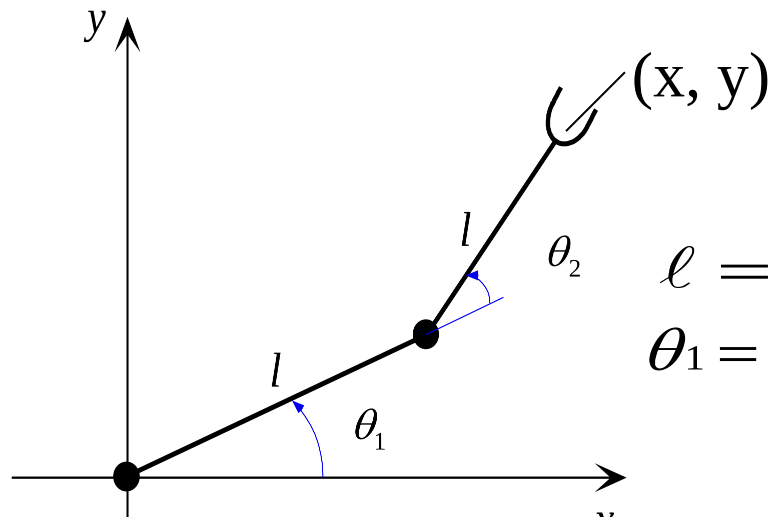


1．ヤコビ行列Jを求めよ.

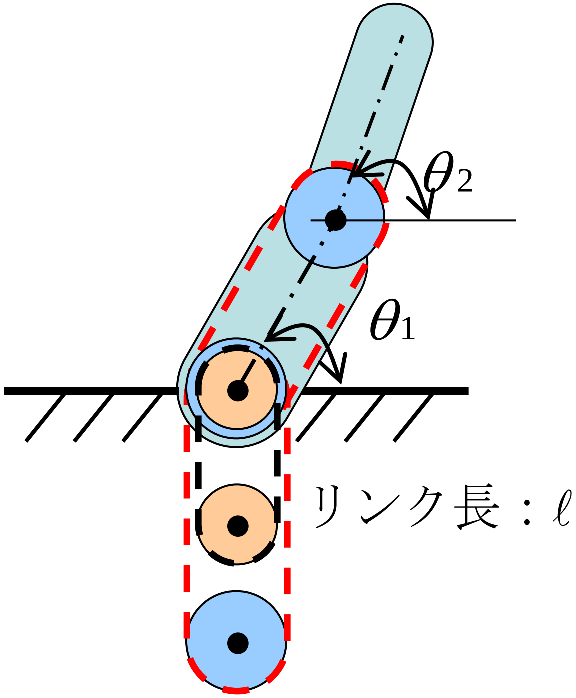
2．特異姿勢を与える (θ_1, θ_2) を求めよ.

＊リンクの幅は無視してよい.

ロボット先端を$(\Delta x, \Delta y)=(0.1, 0.1)$[m]移動させるのに必要な各関節の角度変化 $(\Delta\theta_1, \Delta\theta_2)$ を求めよ.



$$\ell = 1[m]$$
$$\theta_1 = \theta_2 = \pi / 6$$



1．ヤコビ行列Jを求めよ.

2．特異姿勢を与える (θ_1, θ_2) を求めよ.

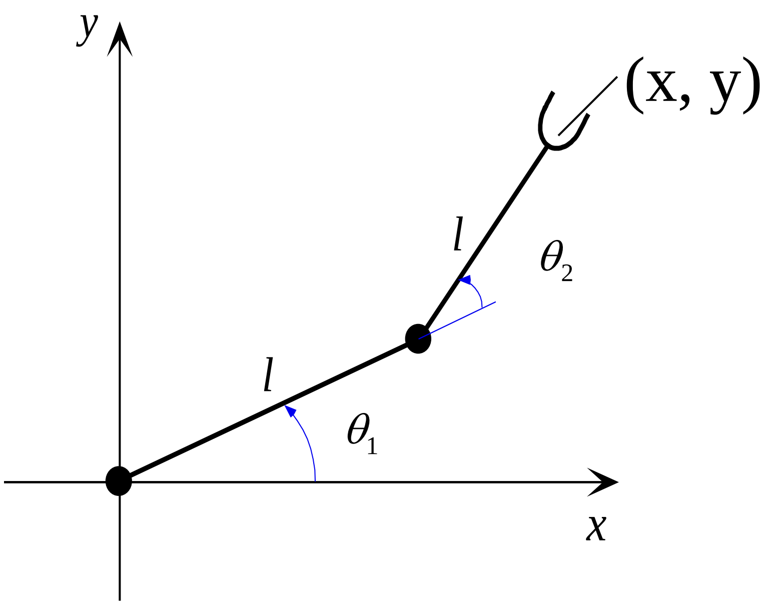
＊リンクの幅は無視してよい.

特異姿勢を与える関係式|J|=0より

$$\sin(\theta_1 - \theta_2) = 0$$

$$\theta_1 = \theta_2 + n\pi \ (n = 0,1,\ldots)$$

ロボット先端を(Δx, Δy)=(0.1, 0.1)[m]移動させるのに必要な各関節の角度変化　$(\Delta\theta_1, \Delta\theta_2)$　を求めよ.



$$\theta_1 = \theta_2 = \pi / 6$$

$$\ell = 1[m]$$

$$d\theta = \mathrm{J}^{-1} d\mathbf{x} \quad (if \ |\mathrm{J}| \neq 0)$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} -ls_1 - ls_{12} & -ls_{12} \\ lc_1 + lc_{12} & lc_{12} \end{pmatrix} \begin{pmatrix} d\theta_1 \\ d\theta_2 \end{pmatrix}$$

$$c_1 = \cos\theta_1, \ \ s_1 = \sin\theta_1$$

$$c_{12} = \cos(\theta_1 + \theta_2), \ \ s_{12} = \sin(\theta_1 + \theta_2)$$

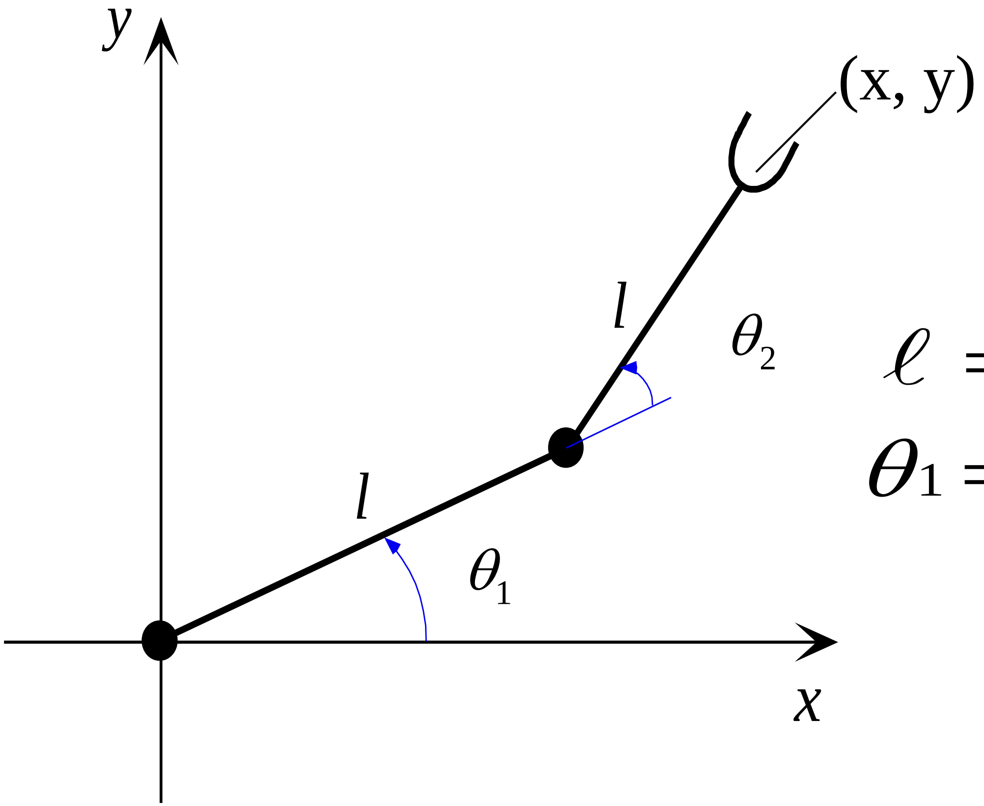
$$\mathrm{J}^{-1} = \frac{1}{|\mathrm{J}|} \begin{pmatrix} lc_{12} & ls_{12} \\ -lc_1 - lc_{12} & -ls_1 - ls_{12} \end{pmatrix}$$

$$|\mathrm{J}| = l^2 \{ -s_1 c_{12} - s_{12} c_{12} + s_{12} c_1 + s_{12} c_{12} \}$$

$$= l^2 \sin\theta_2$$

課題2：2リンクロボットに対して次の問いに答えよ

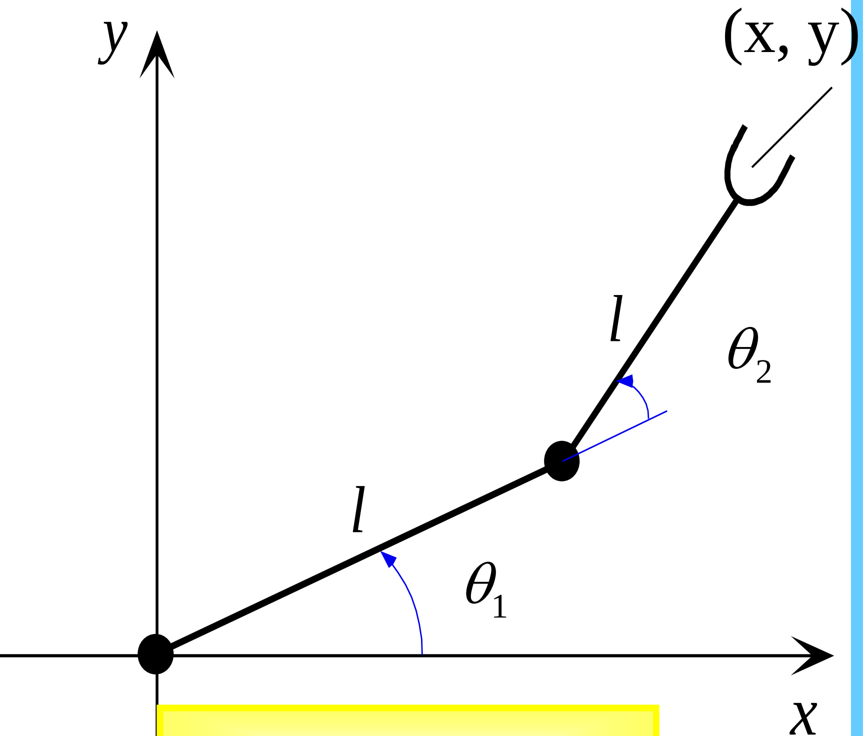
ロボット先端を(Δx, Δy)=(0.1, 0.1)[m]移動させるのに必要な各関節の角度変化 $(\Delta\theta_1, \Delta\theta_2)$を求めよ.



$$\ell = 1[m]$$

$$\theta_1 = \theta_2 = \pi / 6$$

課題2解答



$$\mathbf{dx} = \mathrm{J}d\theta$$

$$d\theta = \mathrm{J}^{-1}d\mathbf{x} \quad (if\ \left|\mathrm{J}\right| \neq 0)$$

$$\begin{pmatrix} dx \\ dy \end{pmatrix} = \begin{pmatrix} -ls_1 - ls_{12} & -ls_{12} \\ lc_1 + lc_{12} & lc_{12} \end{pmatrix} \begin{pmatrix} d\theta_1 \\ d\theta_2 \end{pmatrix}$$

$$c_1 = \cos\theta_1, \quad s_1 = \sin\theta_1$$

$$c_{12} = \cos(\theta_1 + \theta_2), \quad s_{12} = \sin(\theta_1 + \theta_2)$$

$$\mathrm{J}^{-1} = \frac{1}{\left|\mathrm{J}\right|} \begin{pmatrix} lc_{12} & ls_{12} \\ -lc_1 - lc_{12} & -ls_1 - ls_{12} \end{pmatrix}$$

$$\left|\mathrm{J}\right| = l^2 \left\{ -s_1c_{12} - s_{12}c_{12} + s_{12}c_1 + s_{12}c_{12} \right\}$$

$$= l^2 \sin\theta_2$$