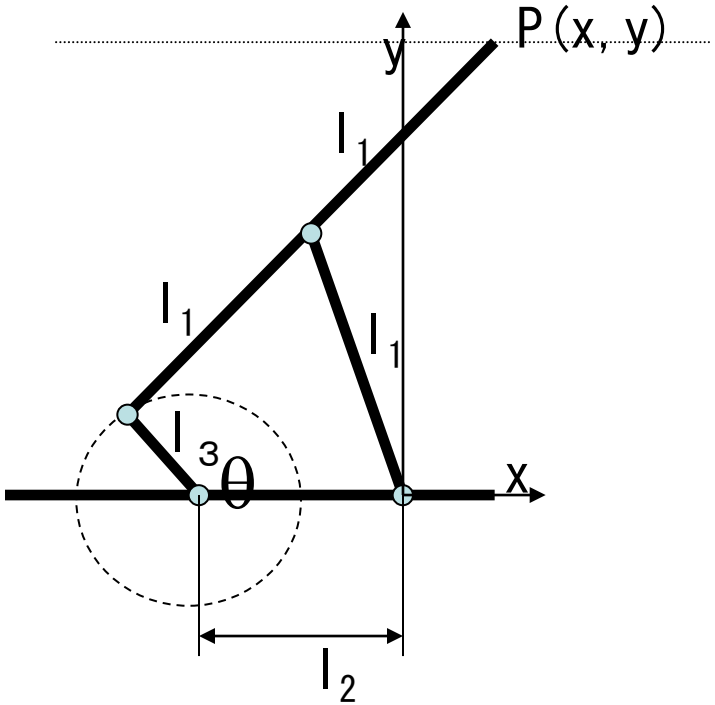
機構学
Part4：リンク機構とその応用

金子真

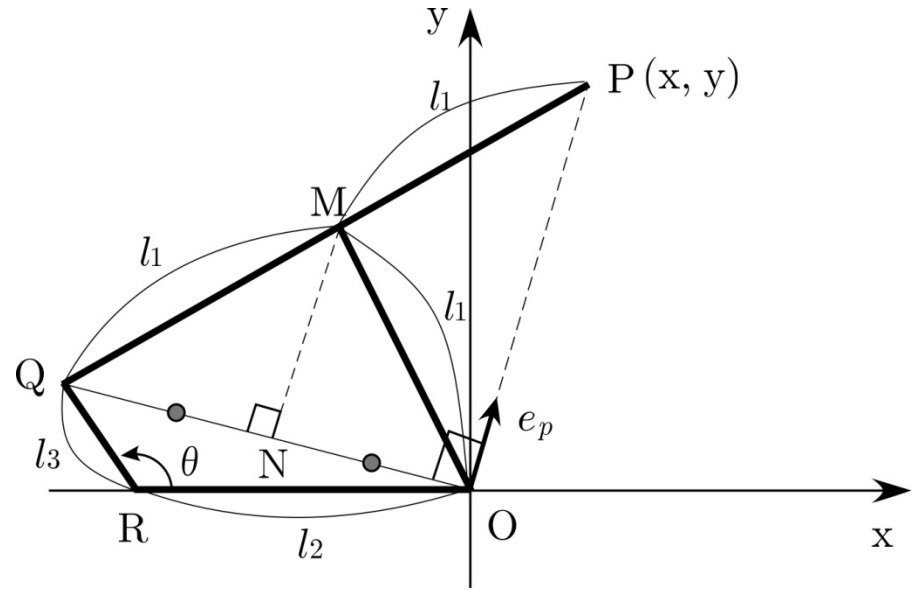
点Pの(x, y)を求めよ.



i)

$$\overrightarrow{OQ} = \begin{pmatrix} l_3\cos\theta - l_2 \\ l_3\sin\theta \end{pmatrix}$$

$$\therefore \overrightarrow{ON} = \frac{1}{2}\begin{pmatrix} l_3\cos\theta - l_2 \\ l_3\sin\theta \end{pmatrix} \quad \text{----------} \quad ①$$



ii)

$$MN^2 = l_1^2 - \frac{1}{4}\left(l_2^2 + l_3^2 - 2l_2l_3\cos\theta\right)$$

$$= \frac{1}{4}\left(4l_1^2 - l_2^2 - l_3^2 + 2l_2l_3\cos\theta\right)$$

$$\overline{PO} = \sqrt{\left(4l_1^2 - l_2^2 - l_3^2 + 2l_2l_3\cos\theta\right)} \quad \text{------} \quad ②$$

iii)

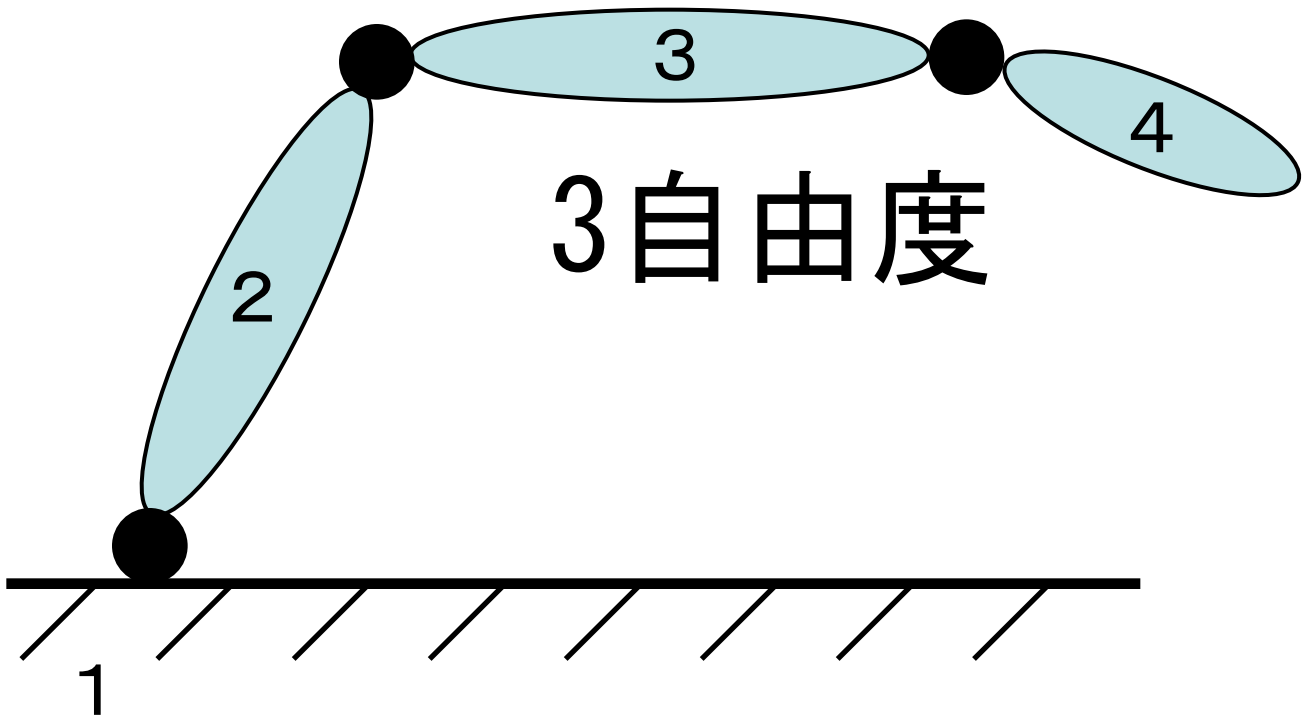
$$\overrightarrow{e_p} = \frac{1}{\sqrt{\left(l_3\cos\theta - l_2\right)^2 + \left(l_3\sin\theta\right)^2}}\begin{pmatrix} l_3\sin\theta \\ -l_3\cos\theta + l_2 \end{pmatrix}$$

$$= \frac{1}{\sqrt{l_2^2 + l_3^2 - 2l_2l_3\cos\theta}}\begin{pmatrix} l_3\sin\theta \\ -l_3\cos\theta + l_2 \end{pmatrix} \quad \text{-----} \quad ③$$

iv)

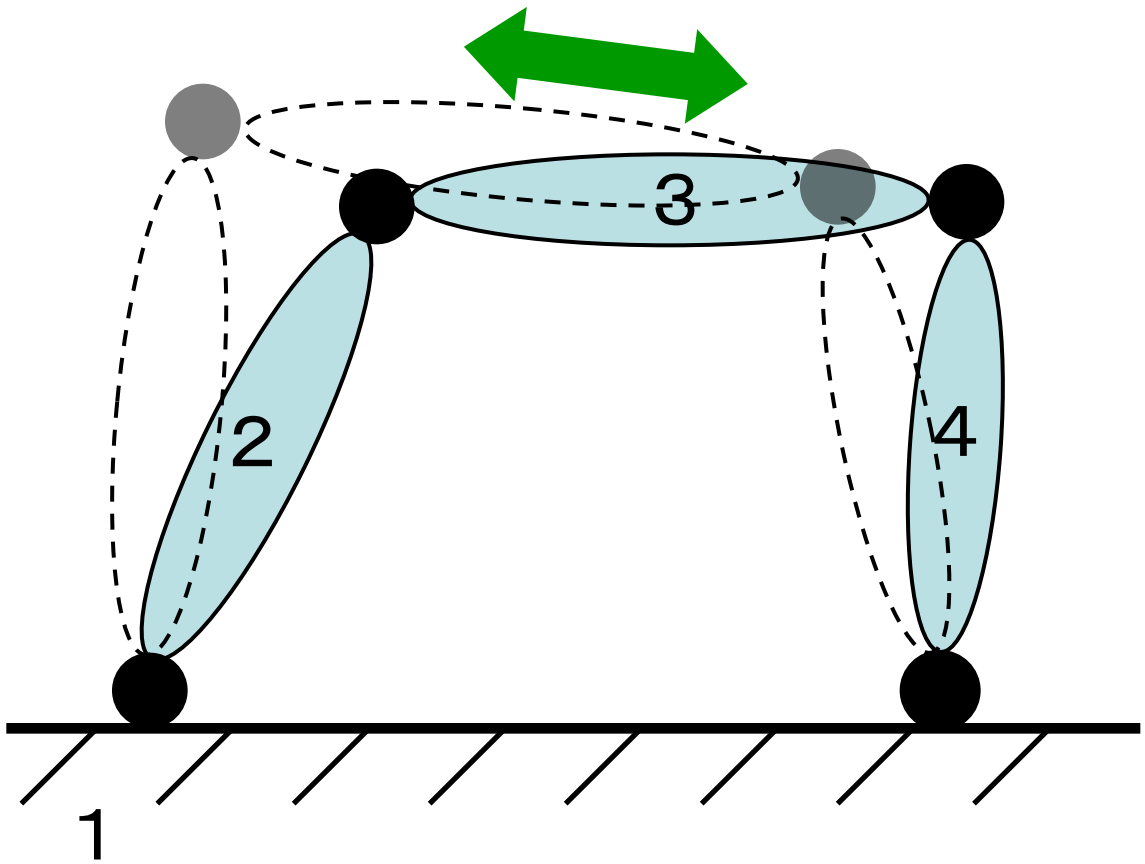
$$\therefore \begin{cases} x = \sqrt{\dfrac{4l_1^2}{l_2^2 + l_3^2 - 2l_2l_3\cos\theta} - 1}\left(l_3\sin\theta\right) \\ y = \sqrt{\dfrac{4l_1^2}{l_2^2 + l_3^2 - 2l_2l_3\cos\theta} - 1}\left(-l_3\cos\theta + l_2\right) \end{cases}$$

開リンク機構の一例

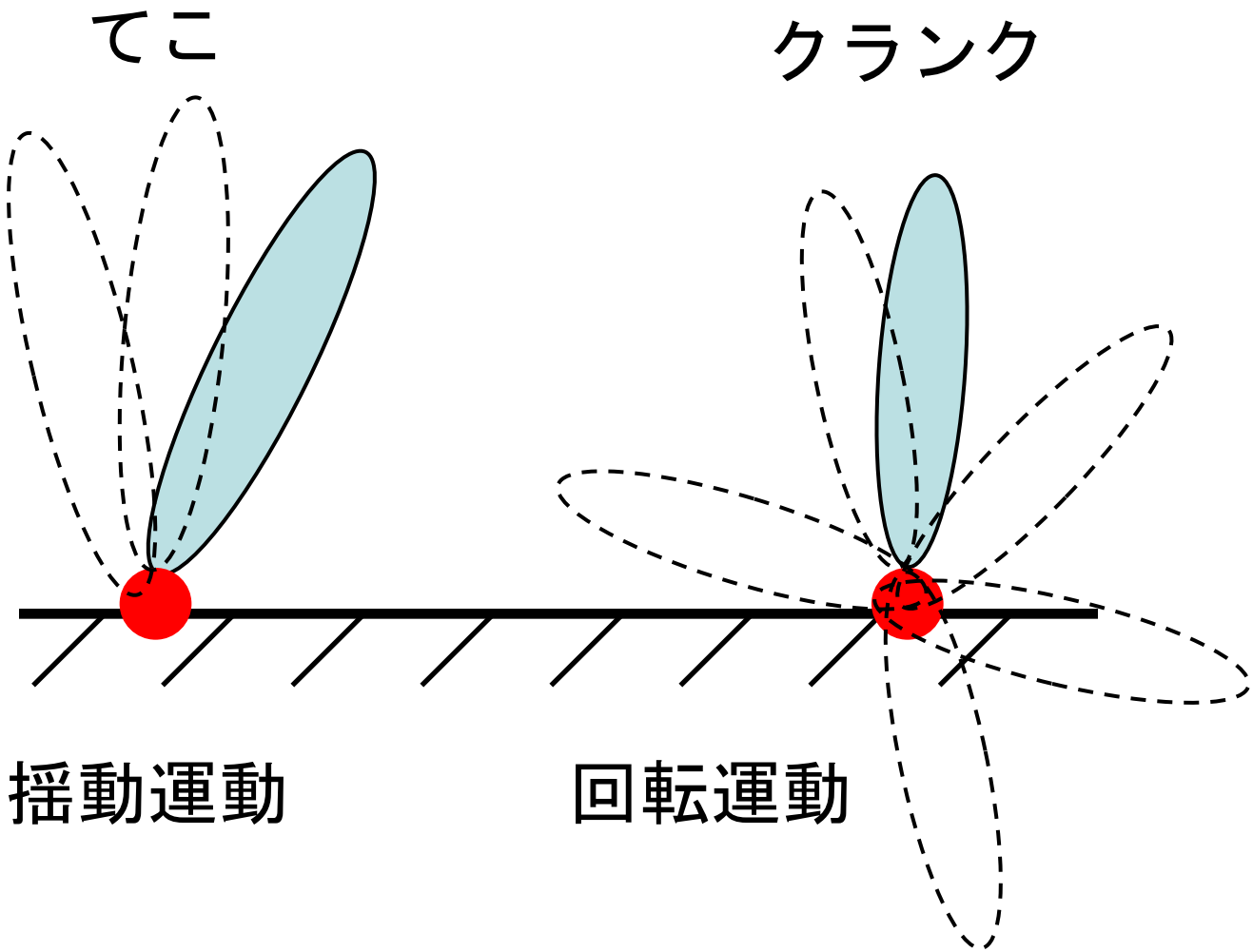


閉リンク機構の一例

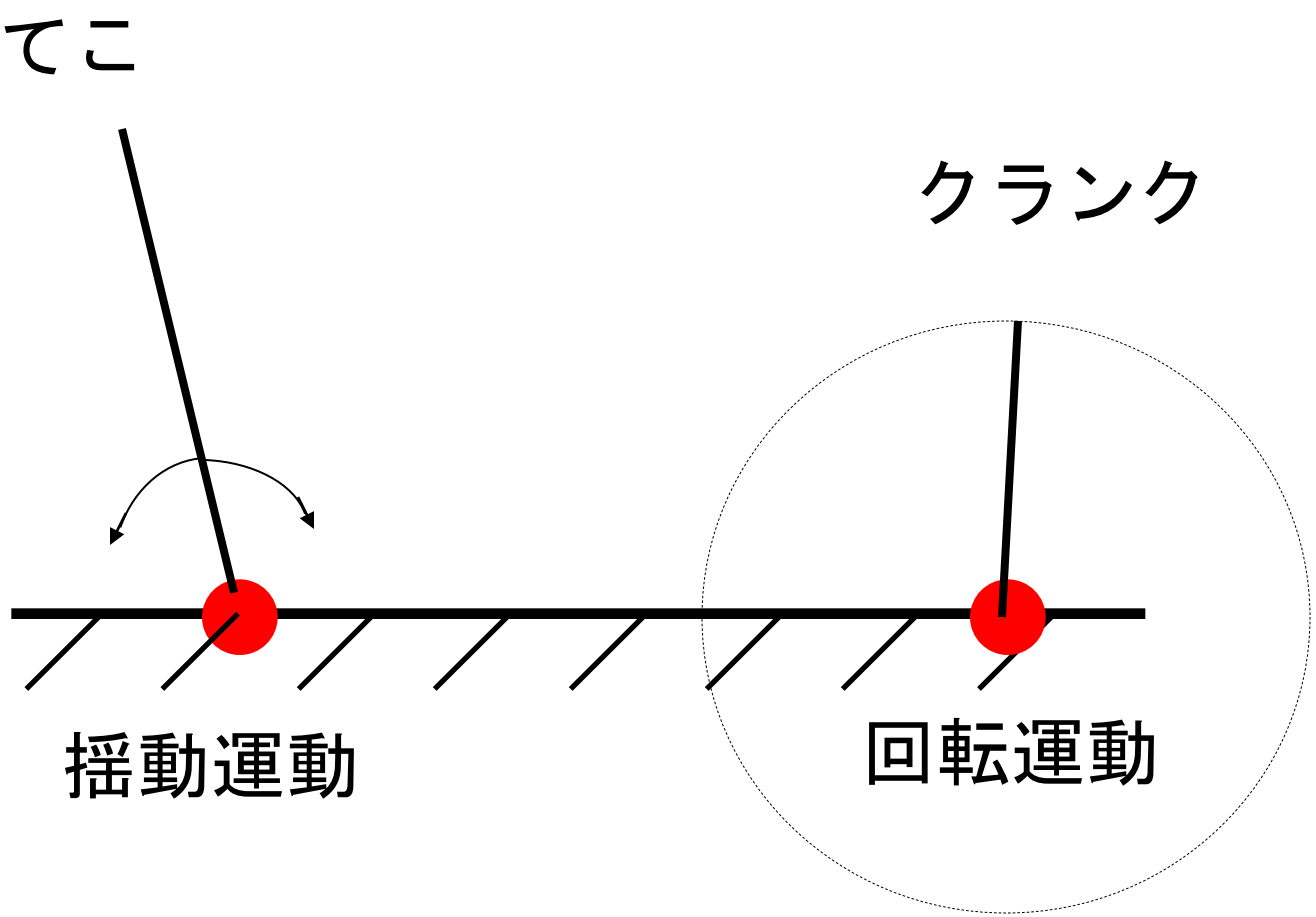
1自由度



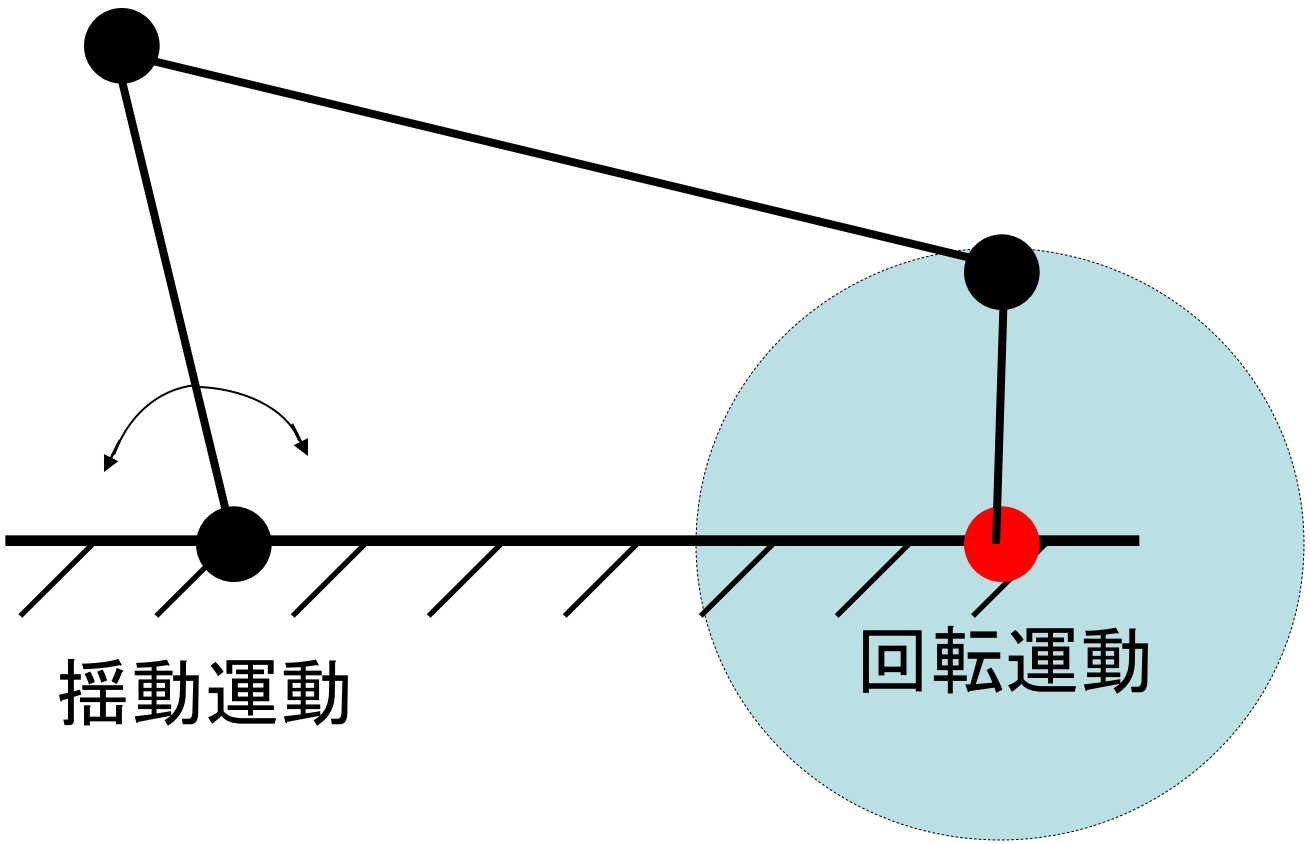
てことクランク



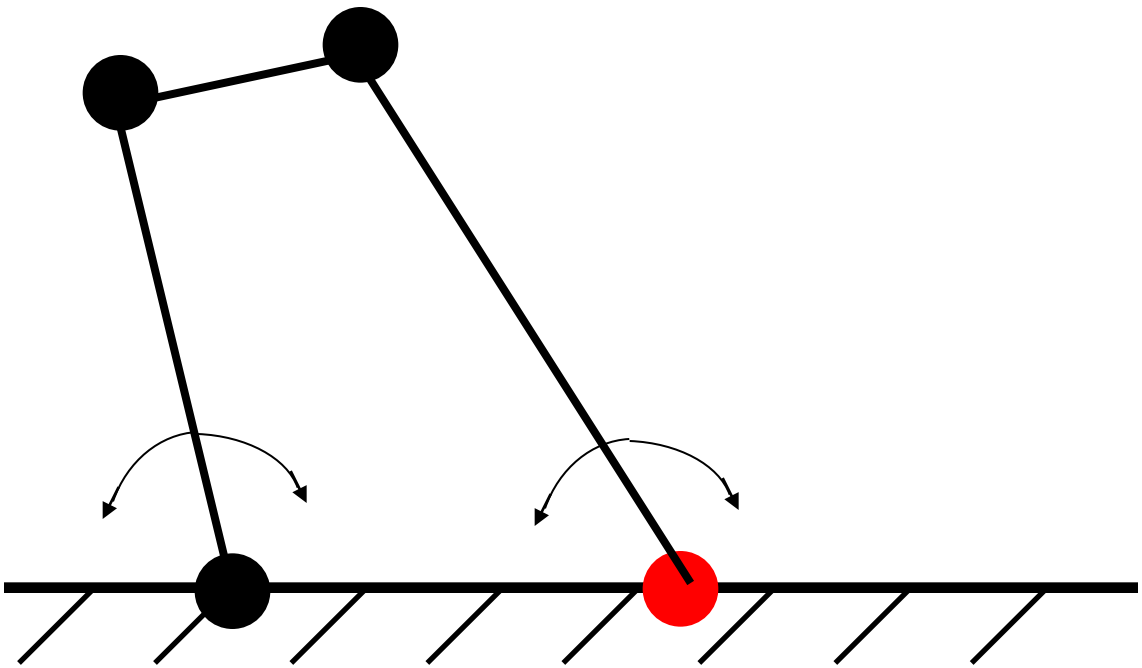
てことクランク



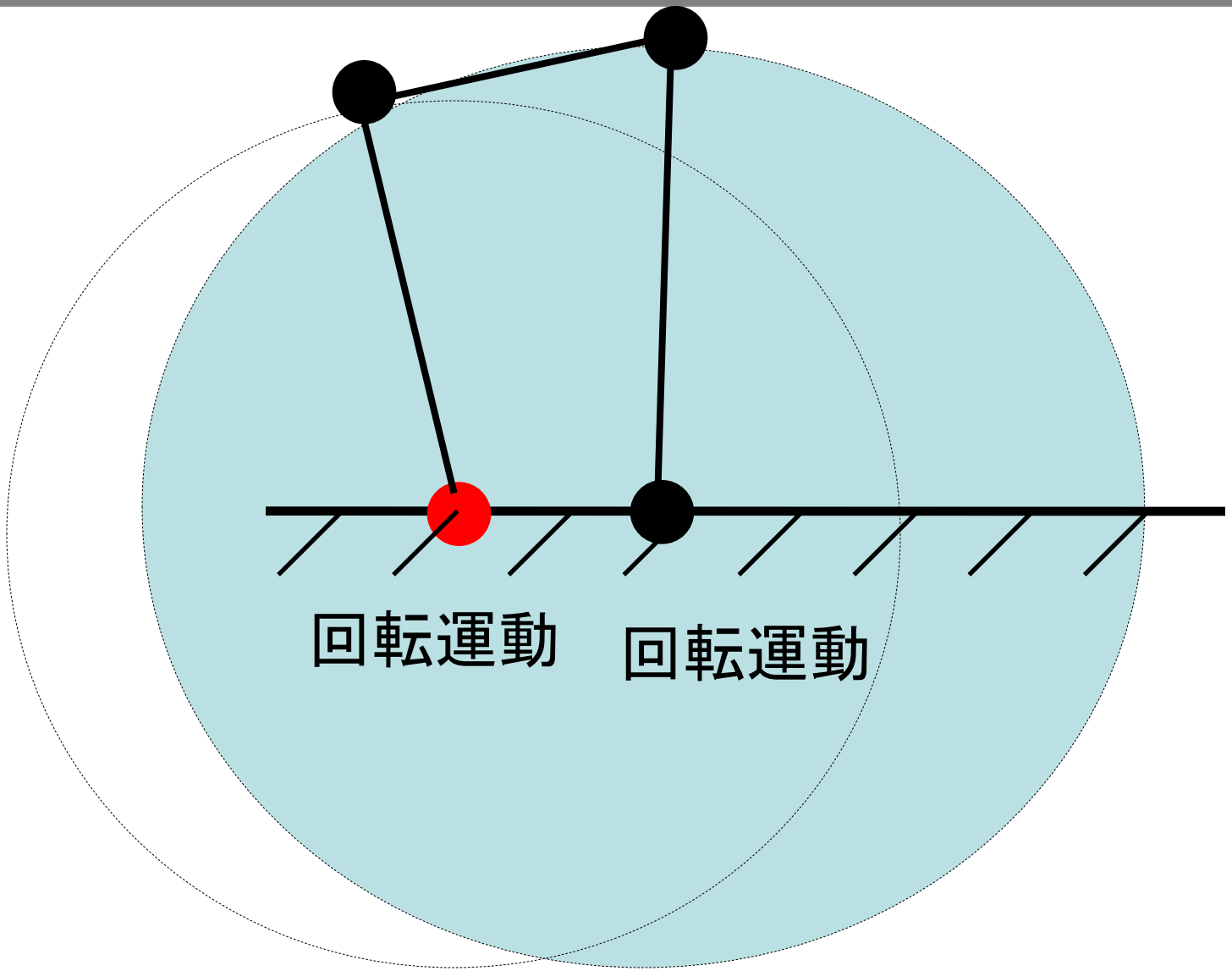
てこ･クランク機構



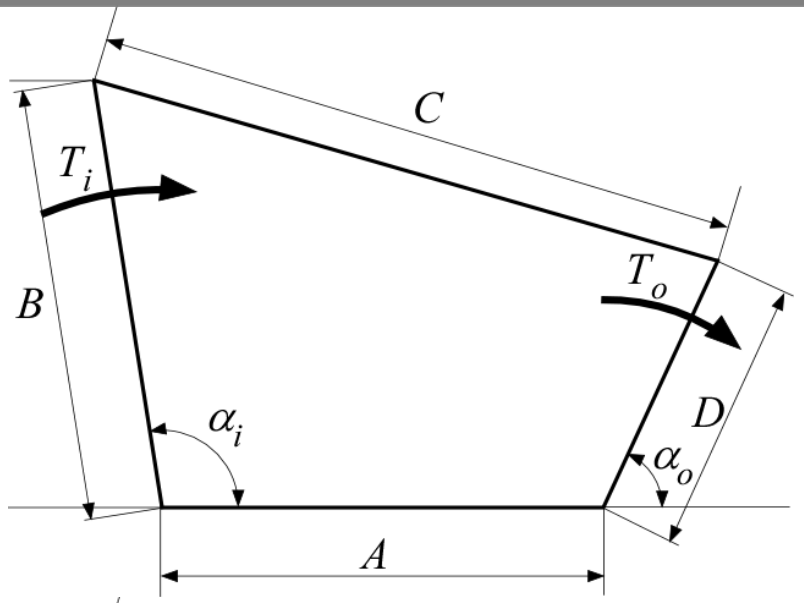
両てこ機構

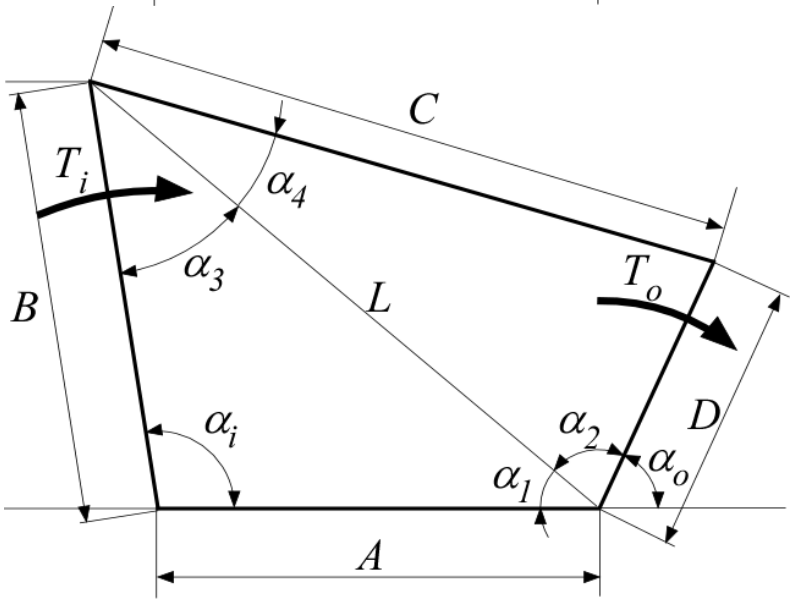


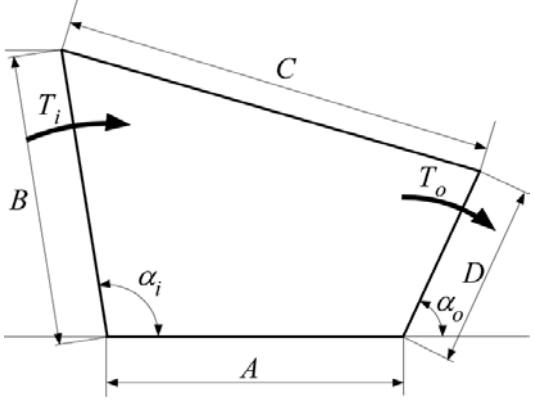
両クランク機構

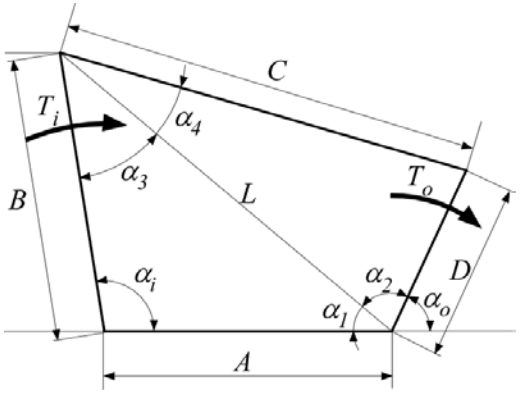


てこ･クランク機構


C
T_i
T_o
B
D
α_i
α_o
A


C
T_i
α_4
α_3
L
T_o
B
D
α_i
α_2
α_o
α_1
A





$$\alpha_1 = \cos^{-1}\left(\frac{A - B\cos\alpha_i}{\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

$$\alpha_2 = \cos^{-1}\left(\frac{A^2 + B^2 - C^2 + D^2 - 2AB\cos\alpha_i}{2D\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

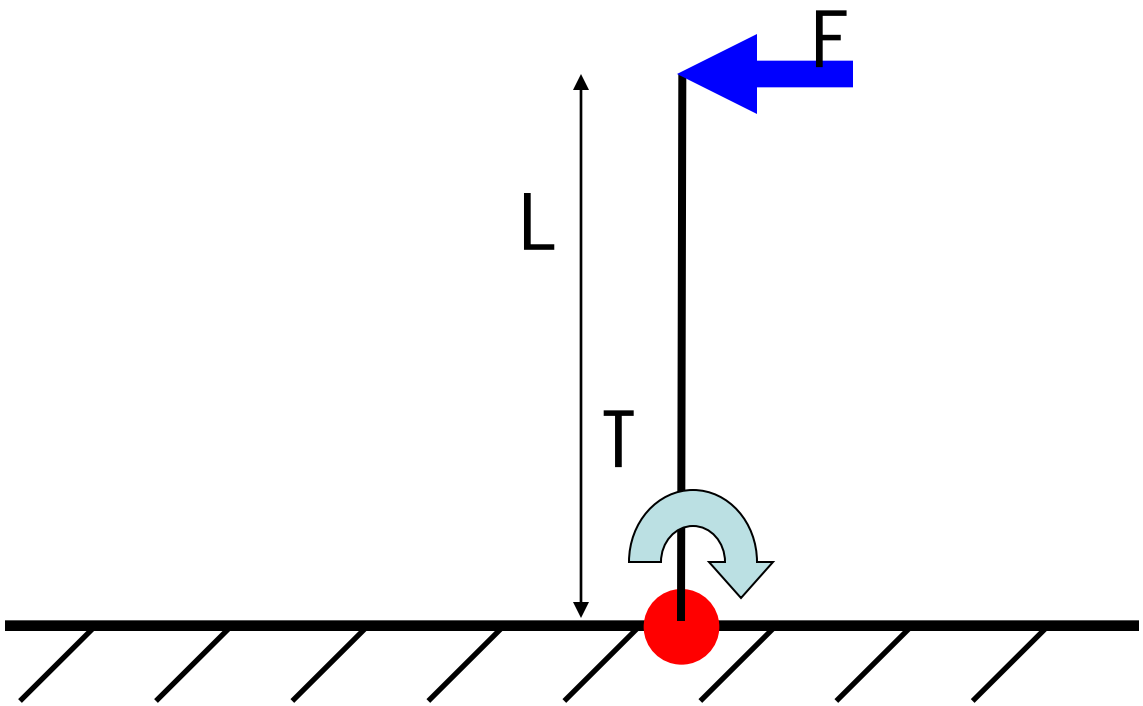
$$\alpha_3 = \cos^{-1}\left(\frac{B - A\cos\alpha_i}{\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

$$\alpha_4 = \cos^{-1}\left(\frac{A^2 + B^2 + C^2 - D^2 - 2AB\cos\alpha_i}{2C\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

$$\alpha_0 = \pi - \alpha_1 - \alpha_2$$

$$T_0 = T_i \frac{D}{B} \frac{\sin(\alpha_2 + \alpha_4)}{\sin(\alpha_3 + \alpha_4)}$$

トルクTと力Fの関係



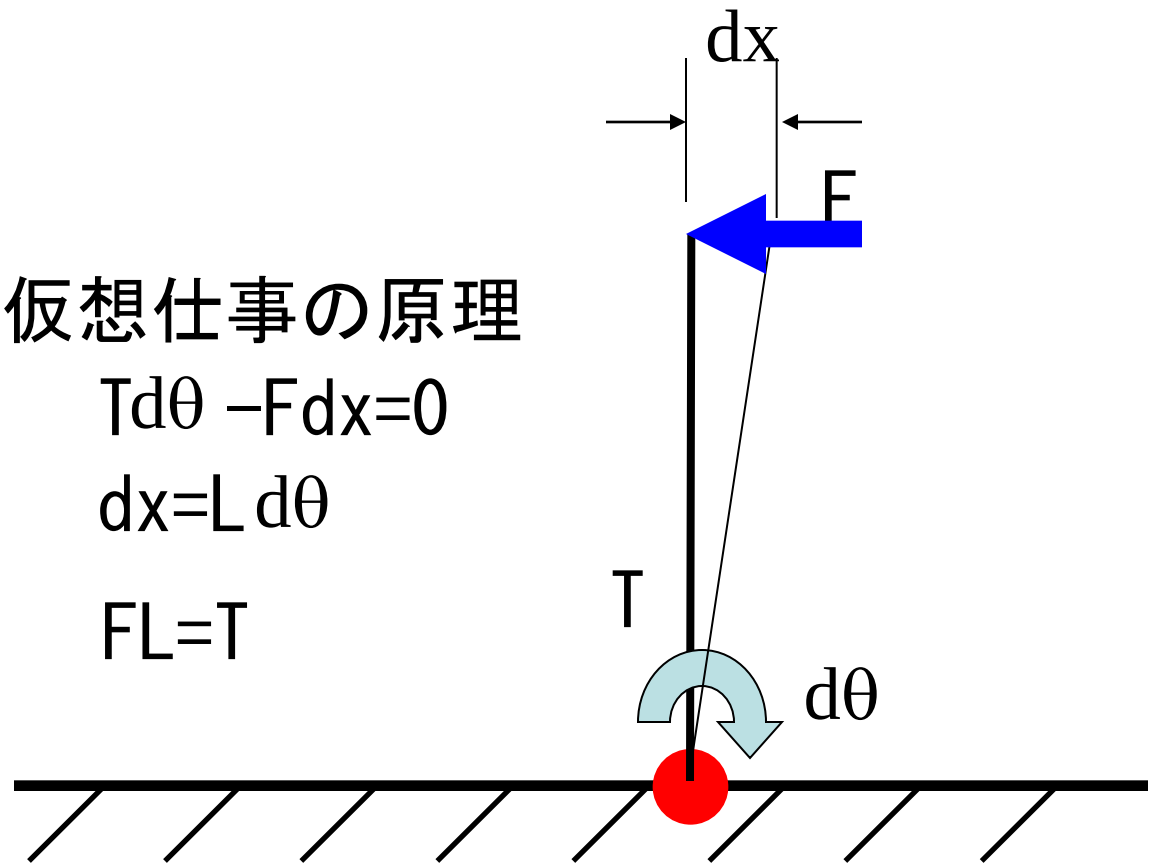
トルクTと力の関係

仮想仕事の原理

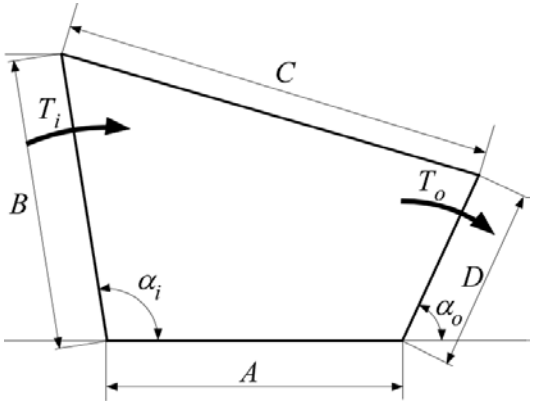
$$T d\theta - F dx = 0$$

$$dx = L\, d\theta$$

$$FL = T$$



仮想仕事の原理



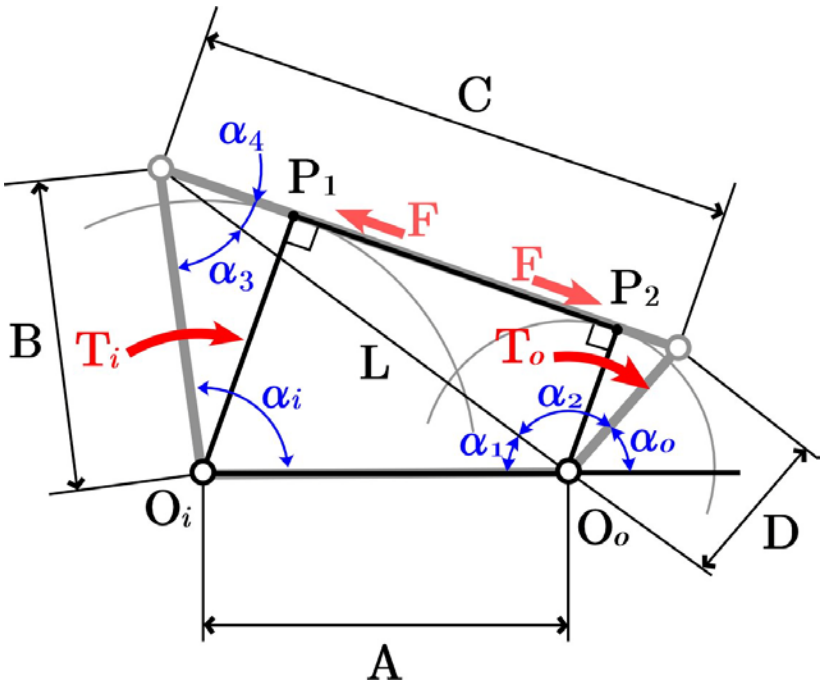
$$T_o d\alpha_o = T_i d\alpha_i$$

$$T_o = T_i \frac{d\alpha_i}{d\alpha_o}$$

Mathematica

$$\frac{T_o}{T_i} = \frac{D\sin(\alpha_2+\alpha_4)}{B\sin(\alpha_3+\alpha_4)}$$

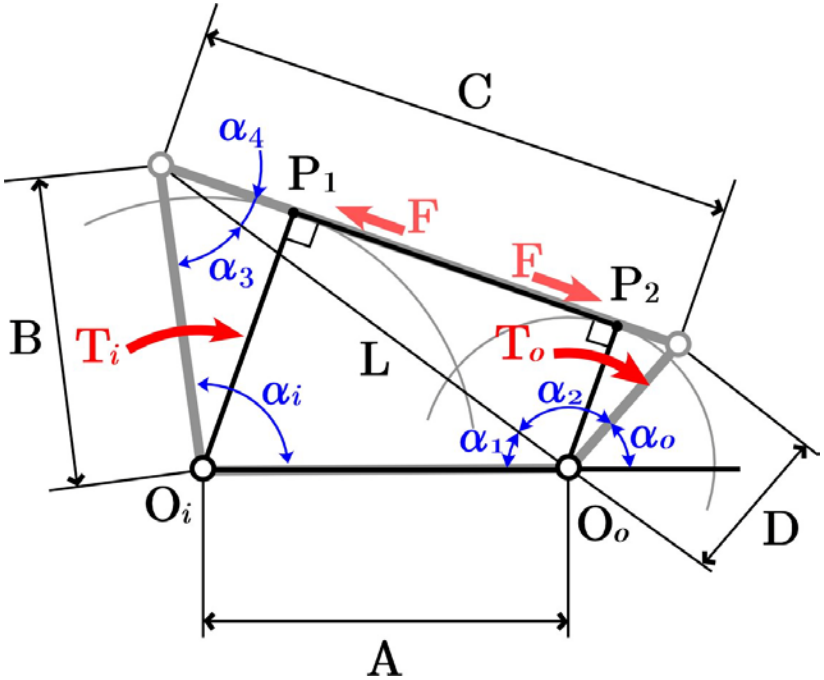
$$\Rightarrow \frac{d\alpha_i}{d\alpha_o} - \frac{D\sin(\alpha_2+\alpha_4)}{B\sin(\alpha_3+\alpha_4)} = ?$$



$$\frac{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}\;\mathrm{Sin}[\alpha i]}{(2\,B-2\,A\,\mathrm{Cos}[\alpha i])\sqrt{\frac{B^2\,\mathrm{Sin}[\alpha i]^2}{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}} - \frac{A\,\mathrm{Sin}\left[\mathrm{ArcCos}\left[\frac{A-B\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]+\mathrm{ArcCos}\left[\frac{B-A\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]\right]}{B\,\mathrm{Sin}\left[2\,\mathrm{ArcCos}\left[\frac{B-A\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]\right]}$$

Mathematica

```
αi = Pi / 3;
A = 0.01;
B = 0.001;
```

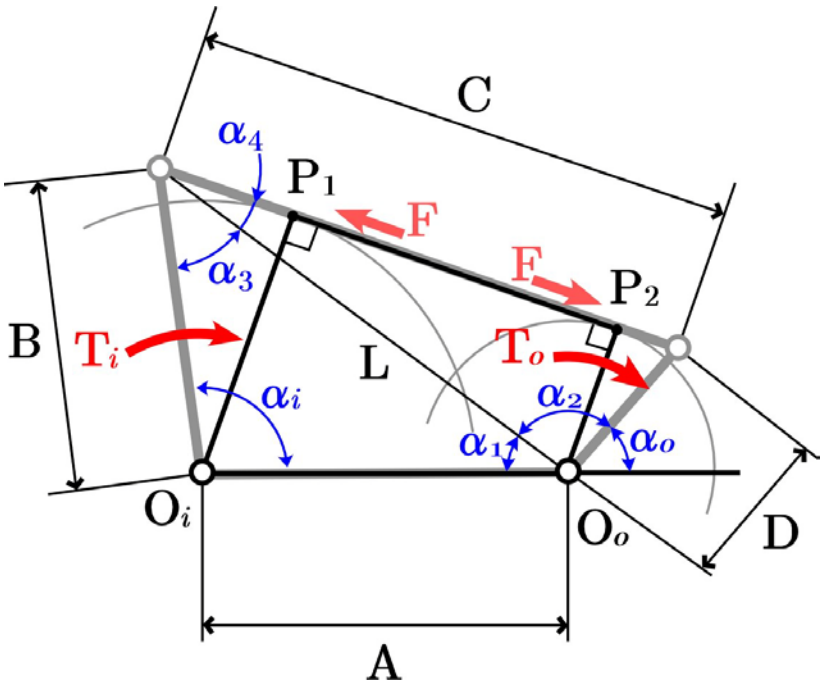


$$\frac{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}\;\mathrm{Sin}[\alpha i]}{(2\,B-2\,A\,\mathrm{Cos}[\alpha i])\sqrt{\frac{B^2\,\mathrm{Sin}[\alpha i]^2}{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}}-\frac{A\,\mathrm{Sin}\left[\mathrm{ArcCos}\left[\frac{A-B\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]+\mathrm{ArcCos}\left[\frac{B-A\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]\right]}{B\,\mathrm{Sin}\left[2\,\mathrm{ArcCos}\left[\frac{B-A\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]\right]}$$

$$= 1.77636\times10^{-15}$$

Mathematica

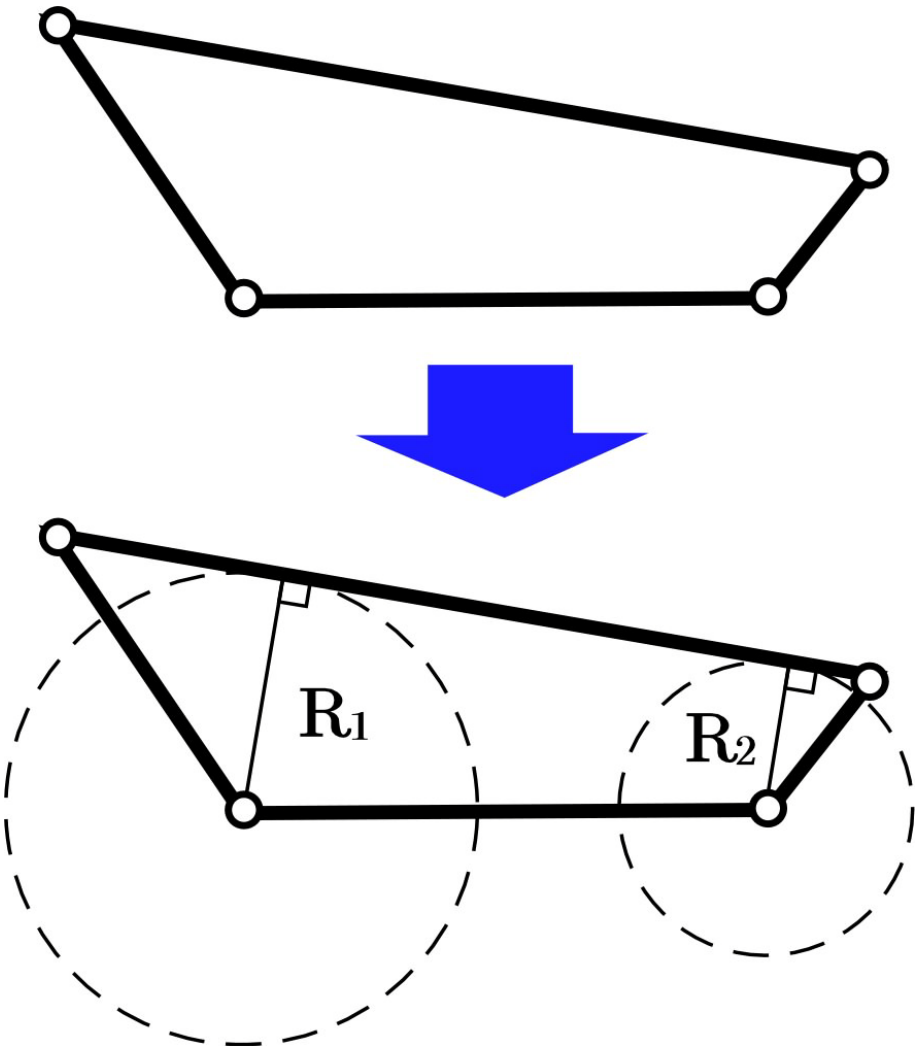
```
αi = Pi / 2;
A = 0.01;
B = 0.001;
```



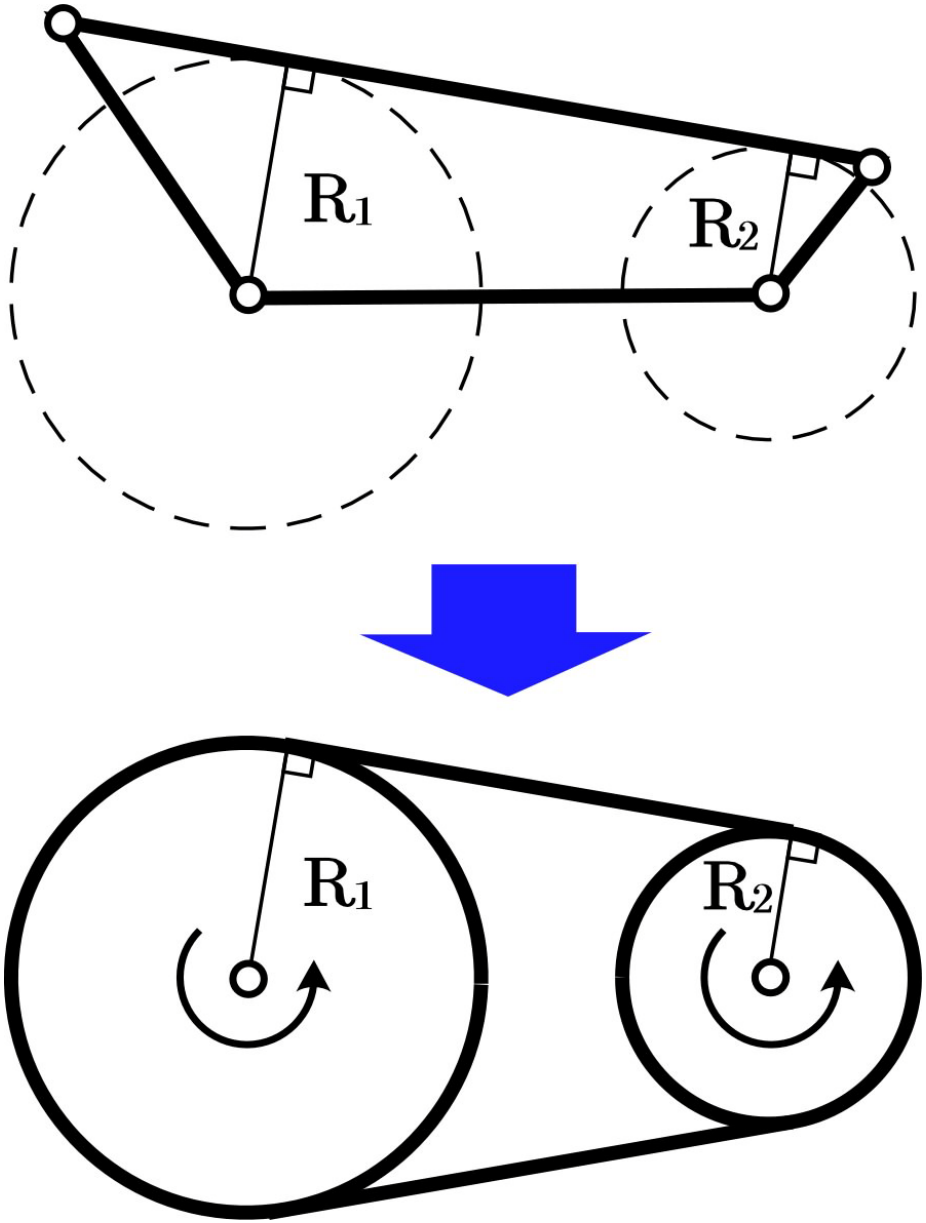
$$\frac{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}\;\mathrm{Sin}[\alpha i]}{(2\,B-2\,A\,\mathrm{Cos}[\alpha i])\sqrt{\frac{B^2\,\mathrm{Sin}[\alpha i]^2}{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}}-\frac{A\,\mathrm{Sin}\left[\mathrm{ArcCos}\left[\frac{A-B\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]+\mathrm{ArcCos}\left[\frac{B-A\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]\right]}{B\,\mathrm{Sin}\left[2\,\mathrm{ArcCos}\left[\frac{B-A\,\mathrm{Cos}[\alpha i]}{\sqrt{A^2+B^2-2\,A\,B\,\mathrm{Cos}[\alpha i]}}\right]\right]}$$

$$= -5.68434\times10^{-14}$$

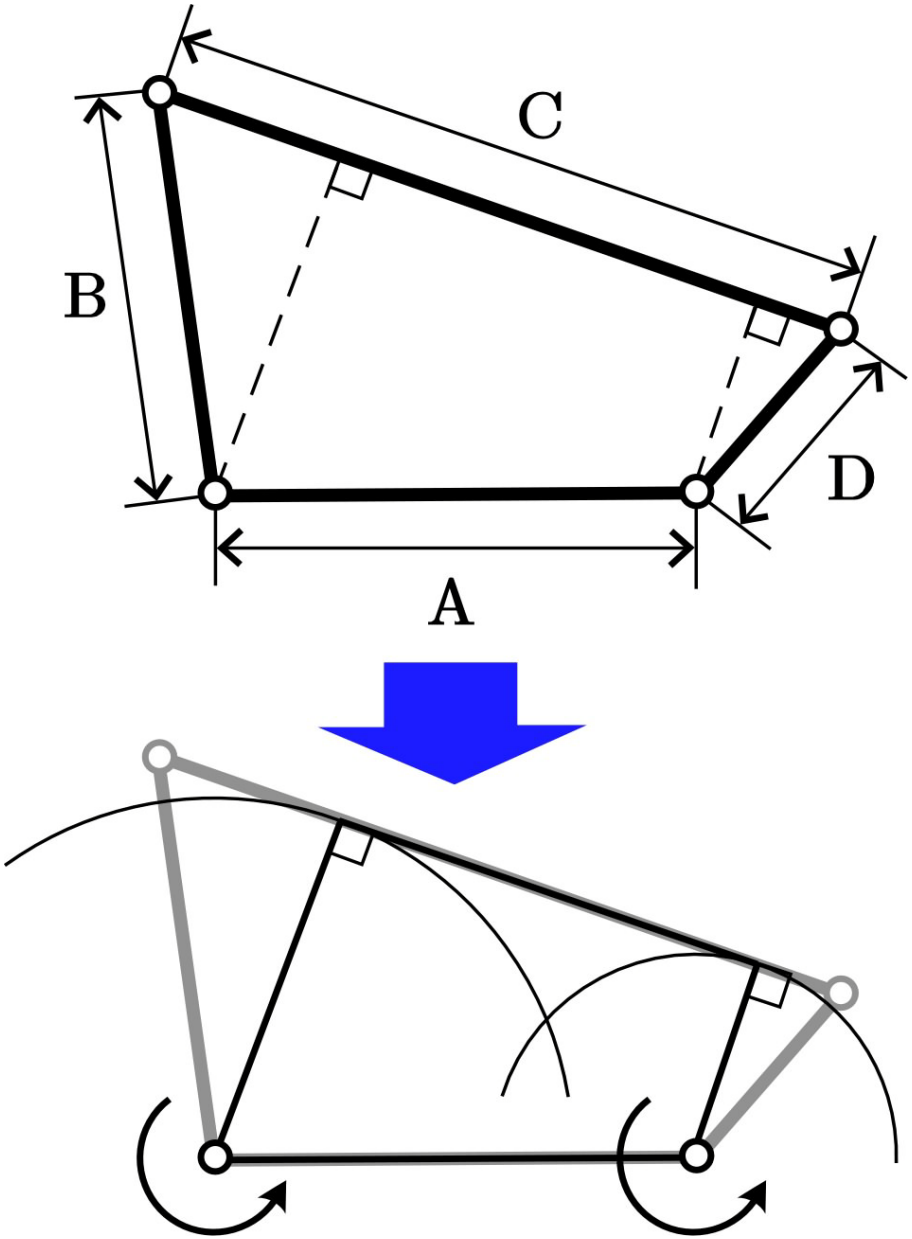
仮想プーリー・ベルトモデル



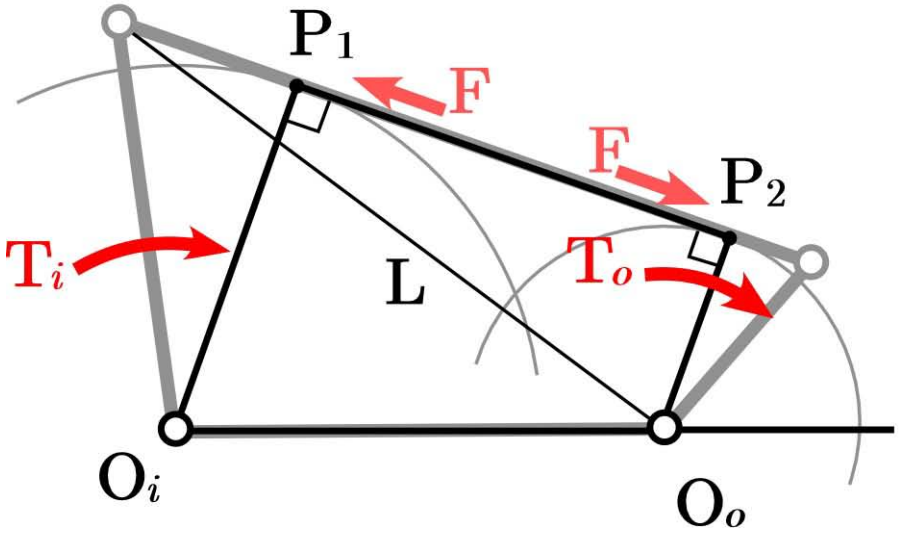
仮想プーリー・ベルトモデル

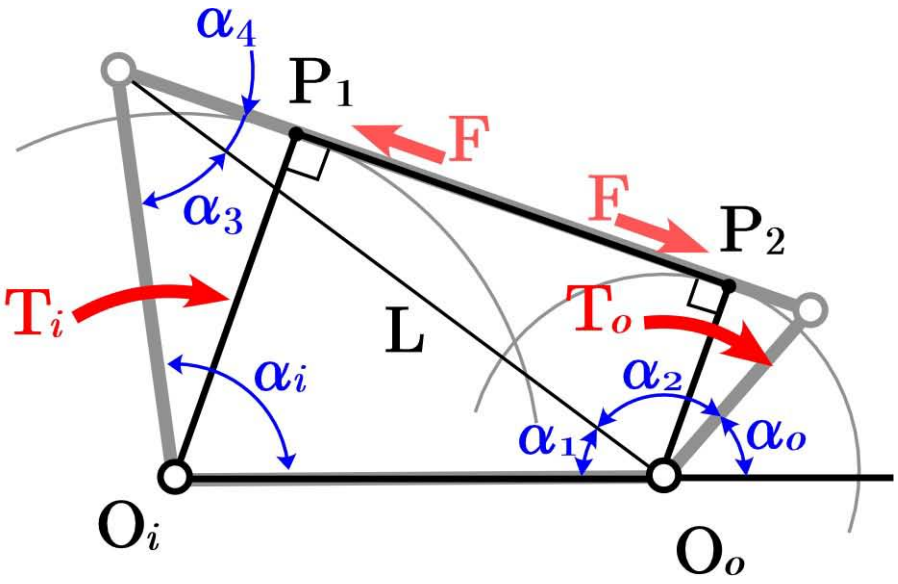


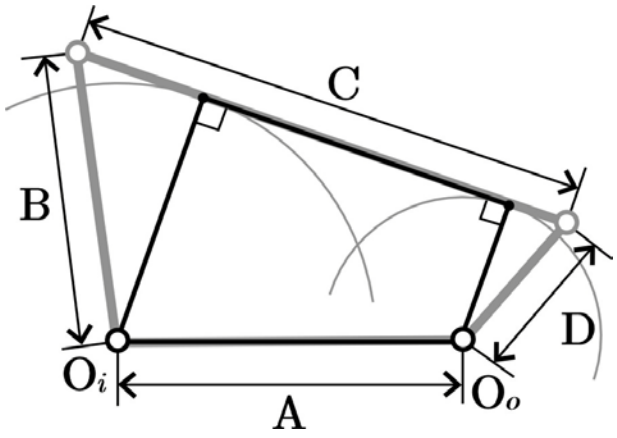
４節リンク機構と等価ベルトモデル



等価モデルによる入出力関係式の導出







$$\overline{O_iP_1} = B\sin(\alpha_3 + \alpha_4)$$

$$\overline{O_oP_2} = D\sin(\pi - \alpha_2 - \alpha_4) = D\sin(\alpha_2 + \alpha_4)$$

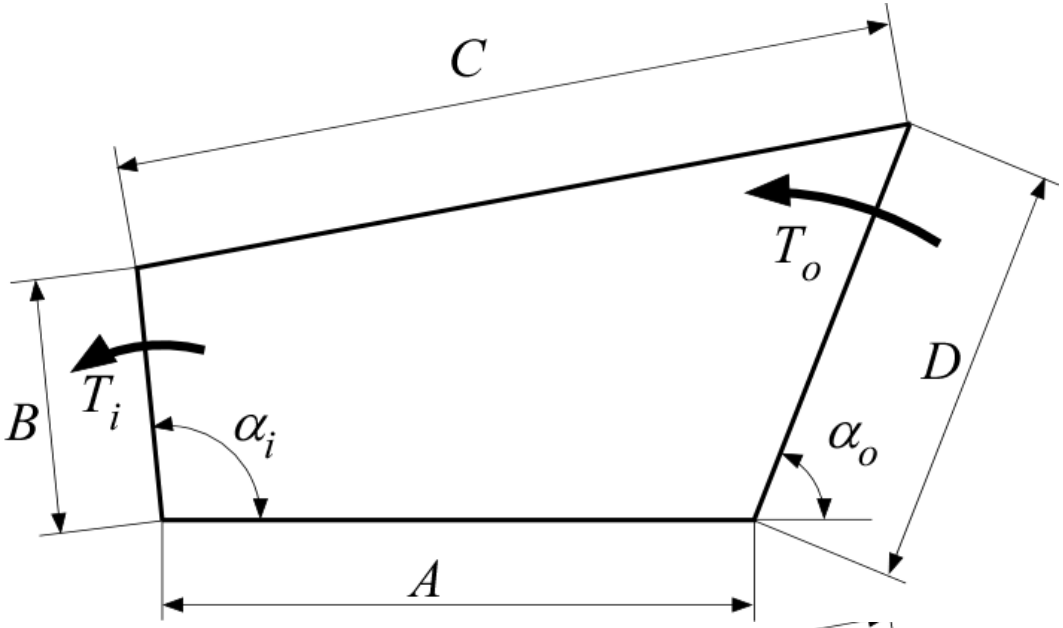
$$T_i = F\overline{O_iP_1}$$

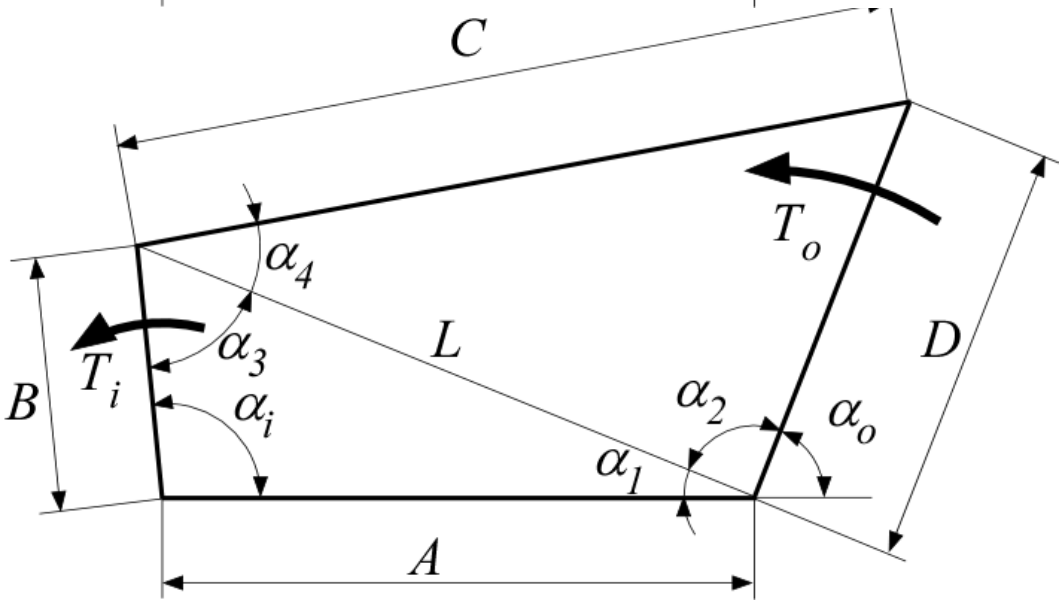
$$T_o = F\overline{O_oP_2}$$

Fを除去すると

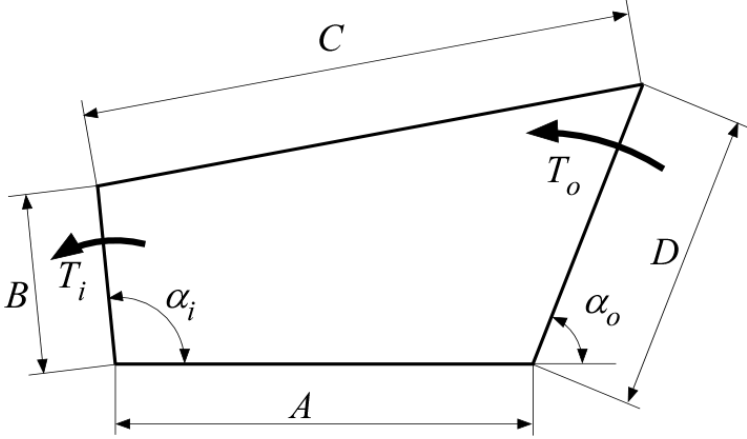
$$\therefore T_o = T_i \frac{D}{B}\frac{\sin(\alpha_2 + \alpha_4)}{\sin(\alpha_3 + \alpha_4)}$$

４節リンクの早送り機構



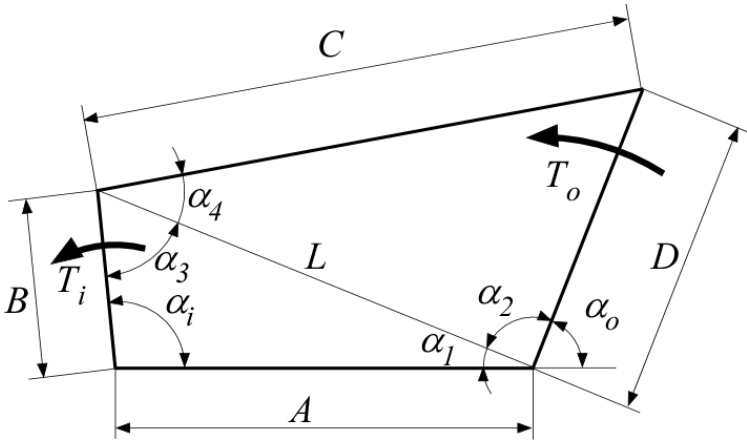


$$\alpha_1 = \cos^{-1}\left(\frac{A - B\cos\alpha_i}{\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$



$$\alpha_2 = \cos^{-1}\left(\frac{A^2 + B^2 - C^2 + D^2 - 2AB\cos\alpha_i}{2D\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

$$\alpha_3 = \cos^{-1}\left(\frac{B - A\cos\alpha_i}{\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

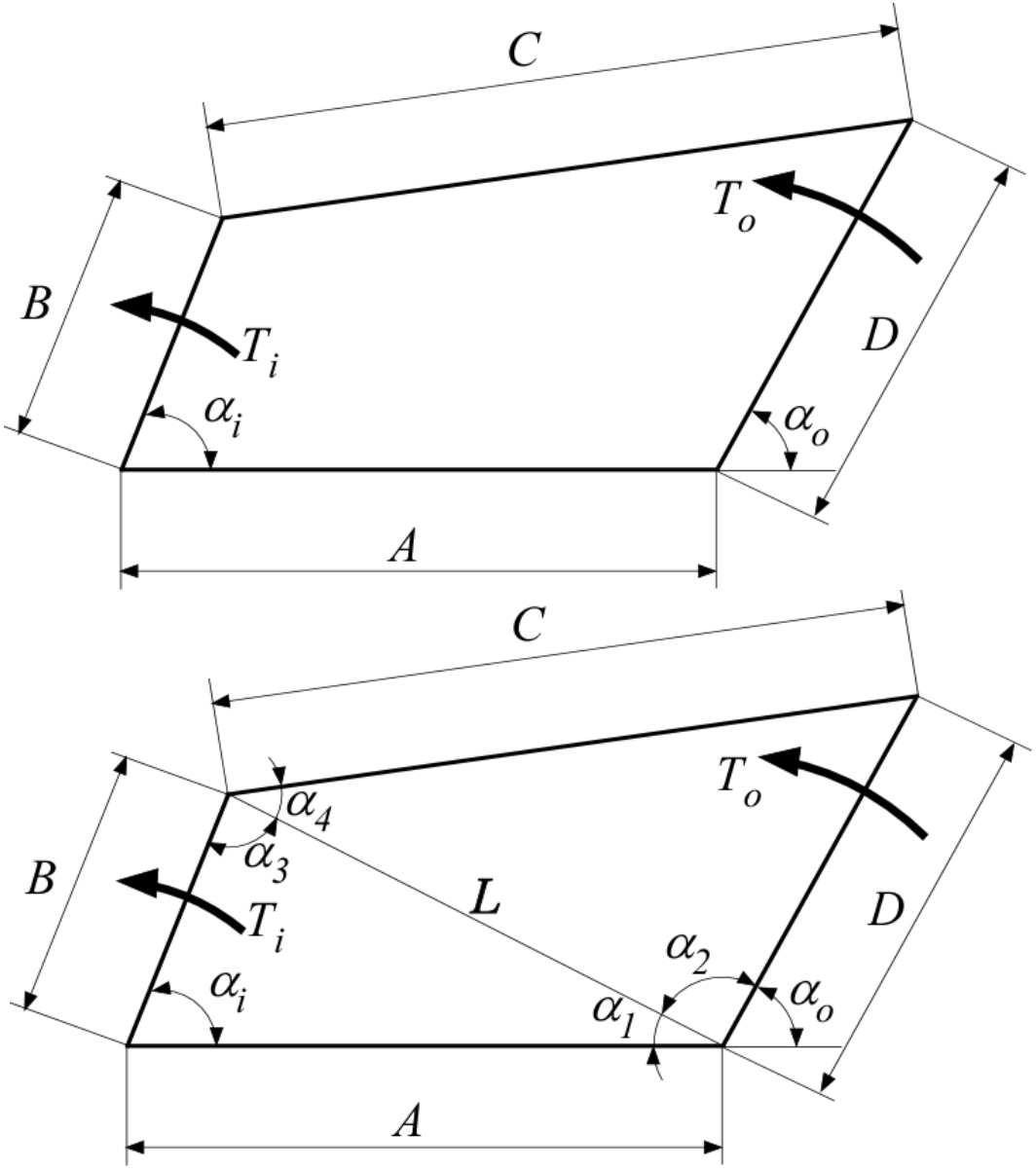


$$\alpha_4 = \cos^{-1}\left(\frac{A^2 + B^2 + C^2 - D^2 - 2AB\cos\alpha_i}{2C\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

$$\alpha_0 = \pi - \alpha_1 - \alpha_2$$

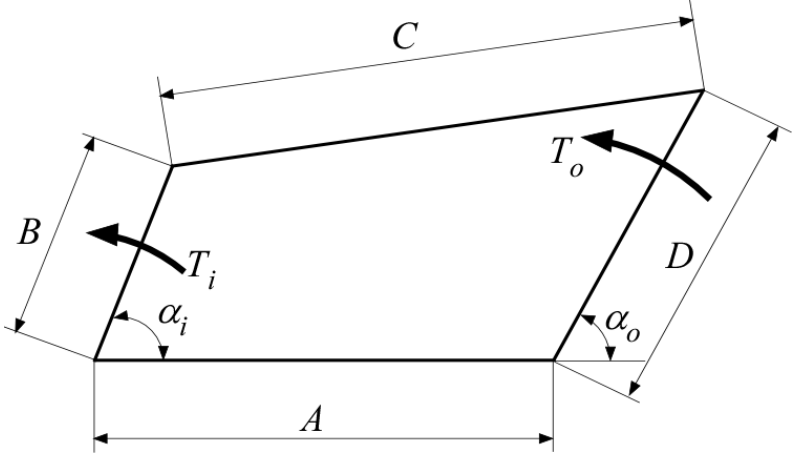
$$T_0 = T_i \frac{D}{B} \frac{\sin(\alpha_2 + \alpha_4)}{\sin(\alpha_3 + \alpha_4)}$$

凹折りたたみてこ・クランク機構



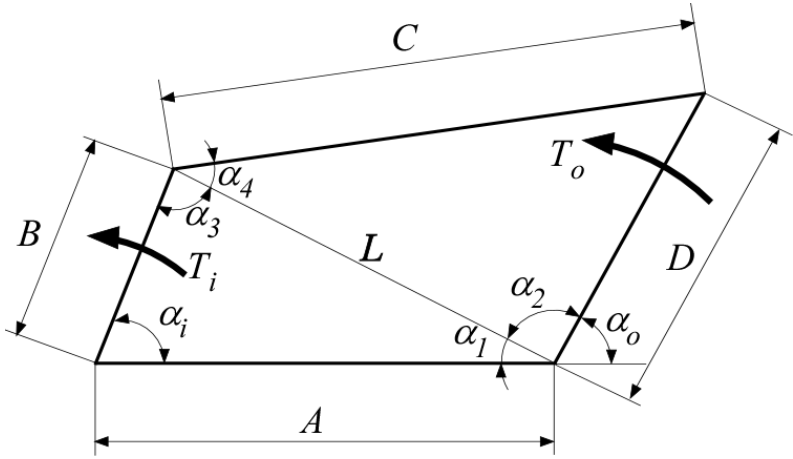
$$\alpha_1 = \cos^{-1}\left(\frac{A - B\cos\alpha_i}{\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

$$\alpha_2 = \cos^{-1}\left(\frac{A^2 + B^2 - C^2 + D^2 - 2AB\cos\alpha_i}{2D\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$



$$\alpha_3 = \cos^{-1}\left(\frac{B - A\cos\alpha_i}{\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$

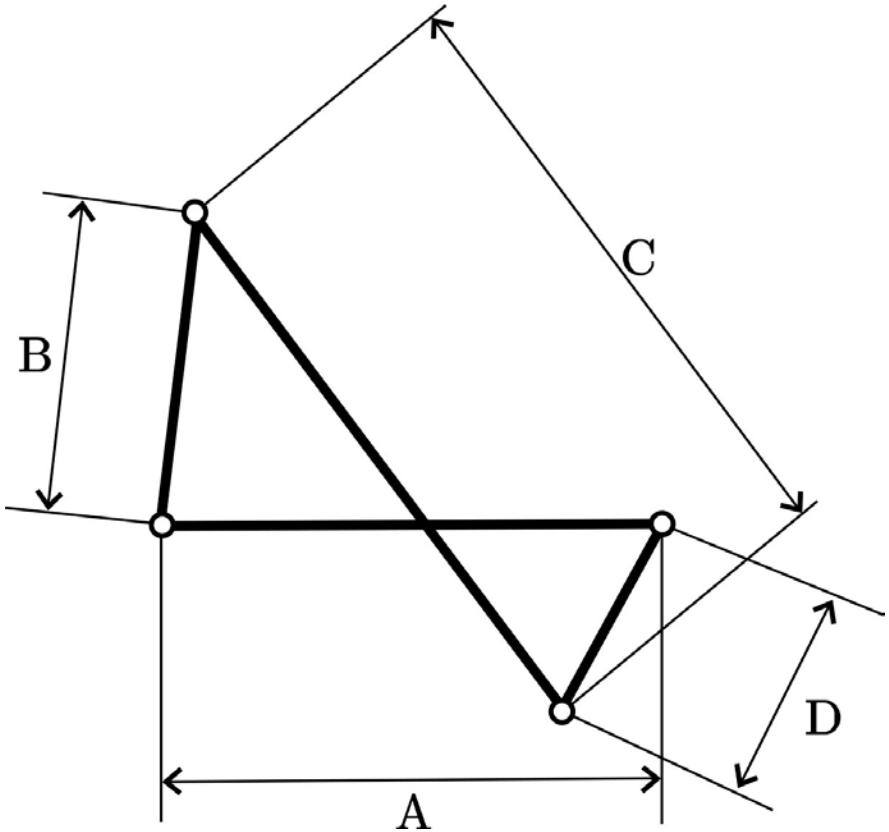
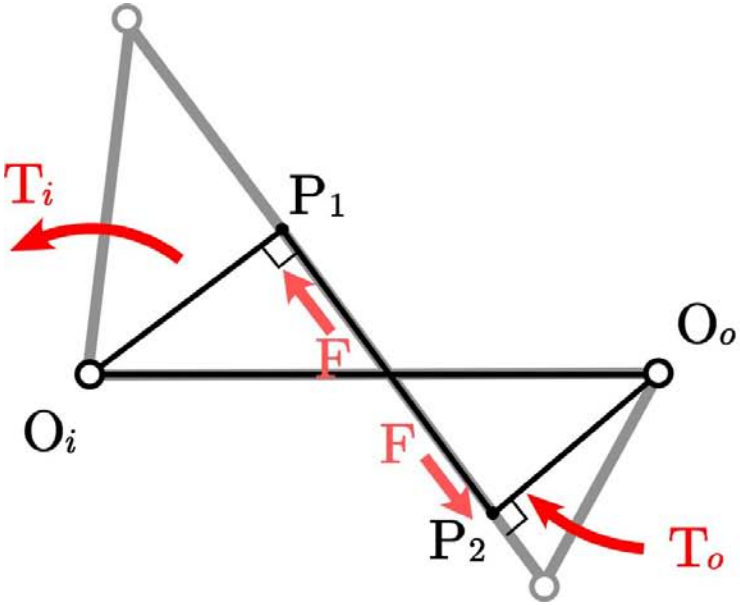
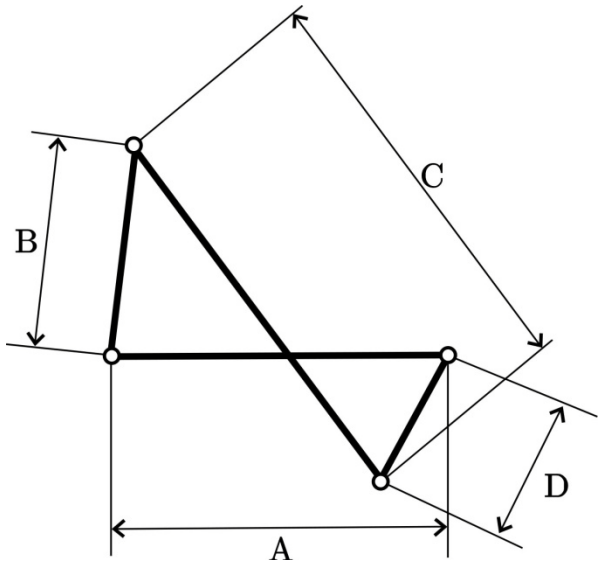
$$\alpha_4 = \cos^{-1}\left(\frac{A^2 + B^2 + C^2 - D^2 - 2AB\cos\alpha_i}{2C\sqrt{A^2 + B^2 - 2AB\cos\alpha_i}}\right)$$



$$\alpha_0 = \pi - \alpha_1 - \alpha_2$$

$$T_0 = T_i \frac{D}{B} \frac{\sin(\alpha_2 + \alpha_4)}{\sin(\alpha_3 + \alpha_4)}$$

等価ベルトモデル



等価ベルトモデル